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Technical Memorandum

CHITRAN - A 7030 (STRETCH) COMPUTER PROGRAM

FOR THE CHI-SQUARE TEST OF NORMALITY

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TECHNICAL MEMORANDUM

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ABSTRACT

The CHITRAN program computes the Chi-square statistic for testing the hypothesis that a random sample is from a normal population. Also given are the degrees of freedom for the Chi-square statistic, a bar chart (frequency distribution), the mean, standard deviation, range, maximum, and the minimum of the observations comprising the sample. Eleven transformations on the observations are possible before the calculations are performed. An optional feature, "pooling," allows the grouping of samples in order to obtain a single Chi-square statistic for the grouped (pooled) data.

CHITRAN is written in FORTRAN IV for the IBM 7030 (STRETCH) computer.

I. INTRODUCTION AND SUMMARY

CHITRAN (Chi-square Test with Transformations) is a program to statistically compare the agreement of the distribution of a sample of observed data with a parent normal distribution having a mean and variance equal to those of the sample distribution.

The distribution of the observations in the sample is classified into a number of intervals (mutually exclusive and exhaustive) and the observed frequency in each interval is then "compared" with the frequency that would be expected in the same interval if the parent distribution were normal. Because the observed and expected frequencies seldom agree, the question is whether the discrepancies are small enough to be considered as due to chance or are so large that the hypothesis of normality must be rejected. The statistic used in CHITRAN to make the "comparison" between observed and expected frequencies, i.e., to test the hypothesis of normality is

$$\hat{\chi}^2 = \sum_{j=1}^{m} \frac{(f_j - \hat{\varphi}_j)^2}{\hat{\varphi}_j} \quad \text{with m-3 degrees of freedom,}$$

where

f_j is the observed frequency in the j^{th} interval $(j=1,2,3,\ldots,m)$

$\hat{\varphi}_j$ is the expected frequency in the j^{th} interval

and m is the number of intervals into which the range of the sample distribution is divided.

The degrees of freedom for the Chi-square statistic are calculated as m-3 because only the general form of the normal distribution can be hypothesized, with the mean and variance being estimated from the observations of the sample. Two degrees of freedom are subtracted from m

because of the estimation of the two parameters and an additional degree of freedom is subtracted because the sum of the expected frequencies is made equal to the number of observed frequencies.

The symbol $\hat{\chi}^2$ is used because, under the hypothesis of normality, this statistic is only approximately distributed as Chi-square (χ^2), with the degree of approximation depending mainly on the size of the expected frequencies. Many authors recommend that $\hat{\varphi}_j$ should be at least 5 for each interval in order that the approximation is adequate. (The minimum number of expected observations in each of the m intervals is an input parameter in CHITRAN and adjacent intervals are combined until this minimum requirement is met.) Assuming that the expected frequency in each interval is large enough that $\hat{\chi}^2$ is approximately distributed as χ^2, the upper tail of the tabled χ^2 distribution can be used as a critical region to test the hypothesis of normality. Examples of the test are given in section V.C.

The Chi-square test for normality, accredited to Karl Pearson in 1900 (with contributions by R. A. Fisher in 1924), is often used even if the the sample is known to be non-normal. For example, physical considerations may restrict the sample observations to non-negative values, but the normal distribution may give a sufficient approximation to the sample distribution over the range of the sample variate. The possibility of approximating a sample distribution by the normal distribution is often investigated because of the characteristics of the normal curve, namely a single mode with the frequencies decreasing symmetrically on both sides of the mode, characteristics which are often found in

physical phenomena. If the normal approximation can be considered adequate, there is a considerable advantage in that the normal distribution is relatively easy to handle mathematically because of the extensive tabulation of the standard normal distribution to be found in the literature.

The numerical value of the Chi-square statistic ($\hat{\chi}^2$), although serving as a criterion to test the hypothesis of normality (or approximate normality) does not indicate the way in which the sample data disagrees with the normal distribution if the hypothesis is rejected. In CHITRAN, therefore, a bar chart of the observed frequencies is printed, which sometimes serves as a visual aid in determining the actual form of the sample distribution if the hypothesis of normality must be rejected. The utility of the bar chart as a visual aid in case of a non-normal distribution can be illustrated by the example problem (section V). Suppose it is not known that the two samples in the example problem are approximately log-normally distributed. Visual inspection of the bar charts of the two samples indicates that the distributions are skewed such that there is a high concentration of small values of the variate and relatively few large values. Since this type of skewness is characteristic of the log-normal distribution, the bar chart should lead the analyst to consider the possibility that the parent populations are distributed log-normally (or approximately log-normally). The example problem also illustrates the aid of the transformations in determining the actual form of a non-normal distribution. If, for example, the hypothesis of normality must be rejected and the bar chart indicates a skewness of the type described above, the Chi-square test

can then be performed on the logarithms of the original values in an attempt to confirm the hypothesis that the parent distribution is actually log-normal.

As many as 500 samples may be processed by CHITRAN with a maximum of 14,000 observations in each sample. Up to 200 "different" Chi-square tests may be performed on each sample of data, where each Chi-square test begins with a different number of intervals, none of which may exceed 400. The capability of performing more than one Chi-square test on a sample of data is included in CHITRAN because the numerical value of the Chi-square statistic and its degrees of freedom are dependent upon the number of intervals into which the sample distribution is divided. Performing several Chi-square tests, each with a different number of intervals, helps protect against the possibility of incorrectly rejecting or incorrectly accepting the hypothesis of normality on the basis of a single Chi-square test. For example, if 10 Chi-square tests (each with a different number of intervals) were performed on the same sample of data, and the hypothesis of normality was found to be acceptable in 9 of the tests and rejected in 1, the multiple testing would minimize the effect of choosing by chance, in a single Chi-square test, that number of intervals which led to rejection.

An optional feature, "pooling," is incorporated into CHITRAN to allow the grouping of samples in order to obtain a single Chi-square statistic for the grouped (pooled) data. This feature was included in CHITRAN primarily for use in analysis of variance, where normality of the residuals in the cells of the classification is essential in the construction of confidence limits and/or tolerance limits and is also

required for tests of significance. CHITRAN can perform the Chi-square test only on the pooled sample or on the pooled sample and the individual samples. (Naturally, the pooled sample option need not be exercised, in which case the Chi-square test is performed only on the individual samples.) The upper limit for the total number of observations in the pooled sample is 1,779,200.

Discussions of the Chi-square test for normality and examples of its application can be found in much of the literature (see, for example, Burr [1953] or Wadsworth and Bryan [1960]). A comprehensive discussion of the test, including its development, application and limitations can be found in Cochran [1952]. Cochran also discusses alternative tests and tests which are supplementary to the Chi-square test.

II. COMPUTATIONAL PROCEDURE

The operations described in this section are performed for each sample of data, after the transformation, or transformations, specified on Card Type 2 (see section III.A.2) have been performed on the n sample observations. If the pooling option has been exercised, the pooled sample is formed as follows before the operations are performed. In each of the samples that are to be pooled, the sample mean is subtracted from the observations in the sample, i.e., the mean of the k^{th} sample is subtracted from each of the n observations of the k^{th} sample. The mean of each individual sample comprising the pooled sample is, therefore, translated to zero. The observations of each sample are then grouped into a single pooled sample (with mean = 0). The computational procedure is described in this section for an individual sample but is also valid for the pooled sample except for the following variations.

The minimum number, INTOVP, of expected observations in each interval of the pooled sample and the number of intervals, INTERP(I), into which the pooled sample frequency distribution is to be divided need not agree with INTOV and INTER(I), the corresponding input parameters for the individual samples. In case of a pooled sample, therefore, the values of INTOVP and INTERP(I) on Card Type 4 must be substituted for INTOV and INTER(I), respectively, in the description of the computational procedure. The only other deviation in procedure for the pooled sample is, as described later, the calculation of the degrees of freedom.

The mean, $\frac{\sum_{i=1}^{n} x_i}{n}$, and the standard deviation, $\sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$, of the n sample observations are computed and printed.

A frequency distribution of the sample observations is then formed in the following way. The maximum, x_{max}, and the minimum, x_{min}, of the observations are determined and the range of the sample frequency distribution is computed as $x_{max} - x_{min}$. In order to obtain the interval width, D, the range is divided by the input value of the desired number of intervals, INTER(I), specified on Card Type 5. The maximum, minimum, range, and the interval width are printed as output. The upper bounds of each of the INTER(I) intervals are then computed by adding 1D, 2D, 3D, ---, j'D, ----, INTER(I)D, respectively, to x_{min}. The maximum, x_{max}, is thereby determined as the upper bound of the last interval. The upper bounds are then printed as output.

NOTE: j' is used to denote the general interval of the INTER(I) intervals into which the sample frequency distribution is divided according to input specifications. j is later used to denote the general interval of the m intervals that result from the restriction

that a minimum number of expected observations must be in each interval. The j'^{th} interval may or may not coincide with the j^{th} interval.

Each observation is then assigned to its proper interval, i.e., to the "first" interval which has an upper bound that is larger than the observation. In other words, the observation x_i is assigned to the j'^{th} interval if x_i is $\geq x_{min} + (j'-1)D$ but $< x_{min} + j'D$. The number of observations in each interval is counted and printed as output under the heading "FREQUENCY." The count in each interval is printed to the right of the upper bound of the interval. A bar chart is then printed in which each observation is represented by an "X".

NOTE: If the number of observations in any interval exceeds 65, the number of X's used in the bar chart to represent the observations is scaled such that only 65 X's are printed for the interval with the largest observed frequency. This scaling is performed in order to preserve the relative proportions of the bar chart, which would otherwise be lost because of printing limitations. The following method is used when scaling is necessary. The maximum observed frequency is divided by 65, giving the number of observations to be represented by each printed X. For example, if the maximum frequency is 98, each X represents $\frac{98}{65} = 1.5077$ observations and the statement "X REPRESENTS 1.5077 OBSERVATIONS" is printed as output. The number of X's to be printed in this case for each

interval is then obtained by dividing the observed frequency in each interval by 1.5077. The number of X's printed is always rounded off to the nearest integer.

After the bar chart has been formed, the quantity $\frac{n}{\text{INTOV}} - 3$ is computed, where INTOV (see Card Type 5, columns 6-10) is the minimum for the expected number of observations to be found in each interval. If $\frac{n}{\text{INTOV}} - 3$ is ≤ 0, the observed frequencies and the bar chart are printed and the statement "CHISQUARE COULD NOT BE COMPUTED" is printed. No attempt is made in this case to compute the Chi-square statistic because of the joint consequence of (a) the restriction that $\hat{\varphi}_j$, the expected number of observations in each interval, must be greater than INTOV and (b) the definition of the degrees of freedom for the Chi-square statistic as the number of intervals, m (for which $\hat{\varphi}_j >$ INTOV), minus 3. If the quantity $\frac{n}{\text{INTOV}} - 3$ is ≤ 0, the degrees of freedom could not be > 0 and further computations would, therefore, be meaningless.

If the quantity $\frac{n}{\text{INTOV}} - 3$ is > 0, an attempt is made to compute the Chi-square statistic. The expected frequency distribution is formed. This distribution gives the number of observations that would be expected in each of the INTER(I) intervals if the sample of n observations was actually from a parent normal distribution having a mean and standard deviation equal to those of the sample observations. Under the hypothesis of normality, the expected frequency ($\hat{\varphi}_j$) in each of the INTER(I) intervals is obtained by multiplying the total number of observations, n, by the probability that an observation will be in a given interval. The probabilities are computed by a system subroutine which uses the standardized normal distribution function,

$$p(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

Consequently, the upper bounds in each interval must be standardized by subtracting the sample mean, $\bar{x}$, and dividing by the sample standard deviation, s. The probability that an observation will be in the first interval is then

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{T_1} e^{-\frac{x^2}{2}} dx,$$

where T_1 is the smallest standardized upper bound. The probability that an observation will be in the second interval is obtained by integrating from T_1 to T_2, where T_2 is the standardized upper bound of the second interval. This procedure is continued until probabilities have been computed for each interval.

Each time an expected frequency, $\hat{\phi}_{j'}$, is found for an interval ($\hat{\phi}_{j'}$ = the number of sample observations multiplied by the probability that an observation is in the j'^{th} interval), the value of $\hat{\phi}_{j'}$ is tested to determine if it is $\leq$ INTOV. If $\hat{\phi}_{j'}$ is $\leq$ INTOV, the expected frequency is added to that of the next interval, i.e., the intervals are "combined." The procedure is continued, if necessary, until a combined interval does have an expected frequency > INTOV. The expected frequencies of succeeding intervals are also examined and, if necessary, combined with the next interval or intervals. Each time an interval (or group of combined intervals) is found to have an expected frequency > INTOV, the expected frequency for the remaining intervals is examined. If INTOV or less expected frequencies remain, the remaining intervals are combined

with the preceding interval. In this way m "new" intervals result from the original INTER(I) intervals, each of the m intervals having an expected frequency, $\hat{\varphi}_j$ $(j=1,2,...,m)$, greater than INTOV.

A count is then made of the number of sample observations, f_j, that actually fall in each of the m intervals and the contribution of each interval to the Chi-square statistic is computed and printed, the contribution of the j^{th} interval being

$$\frac{1}{\hat{\varphi}_j}(f_j-\hat{\varphi}_j)^2.$$

The observed and expected number of observations in each of the m intervals are then printed as output under the headings "OBS FR" and "EXPD FR," respectively.

The quantity m-3, the degrees of freedom, is computed and if ≤ 0, the statement "CHISQUARE COULD NOT BE COMPUTED" is printed. (In case of a pooled sample, the number of degrees of freedom is computed as m-K-2, where K = the number of samples comprising the pooled sample.)

If the number of degrees of freedom is > 0, the Chi-square statistic is computed by summing the contributions of each of the m intervals, i.e.,

$$\hat{\chi}^2 = \sum_{j=1}^{m} \frac{(f_j-\hat{\varphi}_j)^2}{\hat{\varphi}_j}.$$

The value of $\hat{\chi}^2$ is printed as output, along with the degrees of freedom.

III. INPUT PREPARATION

A. Problem Deck Setup

The problem deck is listed below by card type. More than one card of a given type may be necessary for a specific problem. Two sets of Card Type 1 to 7 may be processed at one time, i.e., another problem deck may be stacked behind the first one.

CARD TYPE 1 - VARIABLE FORMAT CARD

CARD TYPE 2 - INPUT CONTROL CARD

CARD TYPE 3 - POOLED SAMPLE IDENTIFICATION CARD (OPTIONAL)*

CARD TYPE 4 - POOLED SAMPLE CONTROL CARD (OPTIONAL)*

CARD TYPE 5 - SAMPLE CONTROL CARD

CARD TYPE 6 - SAMPLE IDENTIFICATION CARD**

CARD TYPE 7 - SAMPLE DATA CARD**

* Omit if samples are not to be pooled.

** This "pair" of Card Types (6 and 7) comprises the data input for one sample. (More than one Card Type 7 may be necessary.) These cards may be input on punched cards or on tape as indicated in column 10 of the INPUT CONTROL CARD (Card Type 2).

1. CARD TYPE 1 - VARIABLE FORMAT CARD

Column	Format	Program Variable	Explanation
1-80	10A8	FØRM	The format specifications by which each Card Type 7 (or data input record, if data is on tape) is to be read. The format specifications must be enclosed by parentheses.

2. CARD TYPE 2 - INPUT CONTROL CARD

Column	Format	Program Variable	Explanation
1-5	I5	NØSAM	The number of samples for which the Chi-square test is to be performed. NØSAM $\leq$ 500.
10	I1	TAPE	2 - The information on Card Types 6 and 7 is on punched cards. 5 - The information on Card Types 6 and 7 is on magnetic tape.
15	I1	GRØUP	0 - Do not pool the samples, i.e., perform the Chi-square test(s) only on the individual sample(s). 1 - Perform the Chi-square tests on the individual samples <u>and</u> on the pooled sample. 2 - Perform the Chi-square test on the pooled sample only. Print the observations comprising the pooled sample. 3 - Perform the Chi-square test on the pooled sample only. Do <u>not</u> print the observations.
19-20	I2	NØTRAN	The number of transformations which are to be performed on all observations comprising each of the samples. The Chi-square test is performed on each transformed set of data. If more than one transformation is desired, the input data must be on tape (i.e., if NØTRAN > 1, then TAPE = 5). X to X is considered to be a transformation, therefore, NØTRAN = 1 if the Chi-square test is to be performed on the original observations only. 1 $\leq$ NØTRAN < 11.

Card Type 2 (Cont'd)

Column	Format	Program Variable	Explanation
21-25	I5	IT(1)	An integer designating the first transformation to be performed on the observations. See the list of 11 available transformations below and the corresponding integer to be entered.
26-30	I5	IT(2)	There are NØTRAN of these entries. If NØTRAN = 2, e.g., the first transformation is specified by an integer in columns 21 to 25 and the second by an integer in columns 26 to 30. The remaining columns are left blank. If only transformation number 6 (the square root) is desired, for example, a six is entered in column 25 and the remaining columns are left blank.
.	.	.	
.	.	.	
.	.	.	
.	.	.	
.	.	.	
.	.	.	
.	.	.	
.	.	.	
.	.	.	
71-75	I5	IT(11)	

Transformations:

The following integers are used to designate the available transformations:

1. $X \leftarrow X$.
2. $X \leftarrow \ln X$.
3. $X \leftarrow \ln[\ln(X)]$.
4. $X \leftarrow \ln(1+X)$.
5. $X \leftarrow [1+\ln(1+X)]$.
6. $X \leftarrow \sqrt{X}$. This transformation is identified as SQRT X in the printout
7. $X \leftarrow \frac{1}{X}$.
8. $X \leftarrow 1+\frac{1}{X}$.
9. $X \leftarrow \sin^{-1}X$. This transformation is identified as ARCSIN(X).
10. $X \leftarrow 2\sin^{-1}\sqrt{X}$. Identified as 2* ARCSIN(SQRT(X)).
11. $X \leftarrow \sin^{-1}\sqrt{X}$. Identified as ARCSIN(SQRT(X)).

3. CARD TYPE 3 - POOLED SAMPLE IDENTIFICATION CARD (OPTIONAL)

This card is omitted if the samples are not pooled, i.e., if column 15 of Card Type 2 = 0.

Column	Format	Program Variable	Explanation
1-80	10A8	PØØLID	Eighty columns to be used for the identification of the pooled sample.

4. CARD TYPE 4 - POOLED SAMPLE CONTROL CARD (OPTIONAL)

This card is omitted if the samples are not pooled.

Column	Format	Program Variable	Explanation
1-5	5X	------	These columns are not read by the program and may, therefore, be used for additional identification of the problem.
6-10	I5	INTØVP	The minimum for the theoretical (expected) number of observations which must be in each of the m intervals used in calculating the Chi-square statistic for the pooled sample. 5 is recommended as a lower limit for this number.
11-15	I5	NØINTP	The number of Chi-square tests to be performed on the pooled sample, where each Chi-square test begins with a different number of intervals, i.e., a different number, INTERP(I). (See below.) NØINTP $\leq$ 100.
16-20	I5	INTERP(1)	The number of intervals into which the range of pooled observations is to be divided for the Chi-square tests. There are NØINTP of these entries.
21-25	I5	INTERP(2)	
.	.	.	
.	.	.	
	I5	INTERP(I)	For example, if three Chi-square tests are desired on the pooled sample (i.e., NØINTP = 3 in column 15), with the range to be divided into 40, 50 and 60 intervals, respectively, then 40 must be entered in columns 19-20, 50 in columns 24-25 and 60 in columns 29-30. The value of INTERP(I) must be $\leq$ 400.
.	.	.	
.	.	.	
.	.	.	
76-80	I5	INTERP(13)	

If NØINTP > 13 but < 30, continue on an additional Card Type 4 in the following manner:

1-5	I5	INTERP(14)
.	.	.
.	.	.
.	.	.
76-80	I5	INTERP(29)

If NØINTP > 29, continue on additional card(s) in the same format as the second Card Type 4.

5. CARD TYPE 5 - SAMPLE CONTROL CARD

There are NØSAM (see columns 1-5, Card Type 2) of these sample control cards, one for each sample of input data. If the number of samples is greater than 1, the sample control cards are stacked consecutively and must be in the same order in which the samples are input on Card Types 6 and 7. If GRØUP = 2 or 3 (column 15, Card Type 2), i.e., if the Chi-square test is to be performed only on the pooled sample, then only NØBS (columns 1-5) and NØINT = 1 (column 15) must be entered on Card Type 5.

Column	Format	Program Variable	Explanation
1-5	I5	NØBS	The number of observations comprising the k^{th} sample (assuming this is the k^{th} sample control card). NØBS ≤ 14,000.
6-10	I5	INTØV	The minimum for the theoretical (expected) number of observations which must be in each of the m intervals used in calculating the Chi-square statistic for the k^{th} sample. 5 is recommended as a lower limit for this number.

Card Type 5 (Cont'd)

Column	Format	Program Variable	Explanation
11-15	I5	NØINT	The number of Chi-square tests to be performed on the k^{th} sample, where each Chi-square test begins with a different number of intervals, i.e., a different number, INTER(I). (See below.) NØINT ≤ 200.
16-20	I5	INTER(1)	The number of intervals into which the range of the observations of the k^{th} sample is to be divided for the Chi-square tests. There are NØINT of these entries.
21-25	I5	INTER(2)	
.	.	.	
.	.	.	
	I5	INTER(I)	For example, if two Chi-square tests are desired on the k^{th} sample (i.e., NØINT = 2 in column 15), with the range to be divided into 10 and 15 intervals, respectively, then 10 must be entered in columns 19-20 and 15 in columns 24-25. The value of INTER(I) must be ≤ 400.
.	.	.	
.	.	.	
.	.	.	
76-80	I5	INTER(13)	

If NØINT > 13 but < 30, continue on an additional Card Type 5 in the following manner:

1-5	I5	INTER(14)
.	.	.
.	.	.
.	.	.
76-80	I5	INTER(29)

If NØINT > 29, continue on additional card(s) in the same format as described above for the second Card Type 5.

6. CARD TYPE 6 - SAMPLE IDENTIFICATION CARD

The data input for each sample consists of a "pair" of card types, the sample identification card (Card Type 6) and the sample data card or cards (Card Type 7). If there is more than one sample, the pair

of card types for sample number 2 follows immediately after those for sample number 1, additional pairs following for each of the remaining samples (if any). The pairs of card types must be in the same order as their corresponding sample control cards (Card Type 5).

Column	Format	Program Variable	Explanation
1-80	10A8	IDENT	Eighty columns to be used for the identification of the observations comprising the k^{th} sample. The observations follow on Card Type 7.

7. CARD TYPE 7 - SAMPLE DATA CARD

Column	Format	Program Variable	Explanation
1-80	Variable	ØBSERV	The numerical values of the observations comprising the k^{th} sample. These values must be input in accordance with the format specified on Card Type 1 (Variable Format Card). If more than one Card Type 7 is necessary for the k^{th} sample, the data is continued on succeeding cards.

B. Job Request Sheets

1401 REQUEST
PRNC-NWL-5230/30 (REV. 3-64)

SECURITY CLASSIFICATION: [ ] TOP SECRET [ ] SECRET [ ] CONF. [X] UNCLASS.

NAME	IDENT NO.	BLDG.	ROOM	TELEPHONE	DATE
R. SHADE	A3	1200	A-143	8361	12-19-66

√	INSTRUCTIONS	√	INSTRUCTIONS
	HOLD TAPE # ________.		PUT ATTACHED DECK ON TAPE # 4156. INPUT PROGRAM [ ] 7090 [ ] 7030
	LIST TAPE # ________.		DUPLICATE AND COMPARE TAPE # ______ ONTO TAPE # ______.
	TYPE DATA [ ] 7030 OUTPUT [ ] 7090 OUTPUT [ ] 7030 BINARY [ ] 7090 BINARY [ ] OTHER ______		UPDATE TAPE # ______ USING TAPE # ______ AND ATTACHED CARDS.
	PAPER WILL BE UNLINED UNLESS ANOTHER TYPE SPECIFIED		COMPILE ATTACHED DECK ON THE 1401. [ ] SPS [ ] AUTOCODER
			OTHER (Continued on reverse)
	PUNCH TAPE # ________. CARD FORMAT: [ ] BINARY [ ] 5081		OPERATOR'S INITIALS / DATE AND TIME COMPLETED

A 1401 request sheet is necessary in order to have Card Types 6 and 7 put on tape. After the data has been put on tape, the proper tape number must be entered on the 7030 job request sheet as shown below. If Card Types 6 and 7 are not on tape but are punched cards in the problem deck, a scratch tape instead of a specific tape should be called for on the 7030 job request sheet.

SECURITY CLASSIFICATION	NAME	IDENT. NO.	ROOM	BLDG.	PHONE	
☐ TS ☐ S. ☐ C. ☒ U.	R. SHADE	A3	A-143	1200	8361	SETUP # ___ OF ___

			JOB CARD								JOB TITLE		
☐ COMPILE	☒ GO	☐ CK OUT	CHARGE CODE				IDENT.		PROGRAMMER				
☐ COMPILGO		☒ PROD	2	1	5	8	C	T	A	3	EST. COMPILER TIME: ---------	EST. EXECUTION TIME: 1 MIN	DATE: 12-19-66

TAPES CALLED FOR BY PROBLEM PROGRAM

TAPE NUMBER	4156									
FILE PROTECT ON	Yes									
PROGRAMMER NUMBER										
SPECIAL HANDLING (*See attached Inst.*)										

OPERATOR'S COMMENTS	WAIT FOR TAPES
☐ ABEOJ ☐ HOLD ☐ LOOP ☐ IF CHECKED, SEE REVERSE SIDE FOR ADDITIONAL COMMENTS.	

SPECIAL INSTRUCTIONS (*continued on reverse*)

7090 JOB REQUEST PRNC-NWL-5230/29 (REV. 2-63)

The number of the tape containing the data on Card Types 6 and 7 must be punched in the IOD deck (for example, 4156 is punched in place of XXXX on the "REEL, PULXXXX" card of the IOD deck if Card Types 6 and 7 are on tape 4156).

IV. FORMULATION OF OUTPUT

(Identification of sample as given on Card Type 3 or Card Type 6)

CHI SQUARE TEST ___ WITH ___ OBSERVATIONS AND ___ INTERVALS. TRANSFORMATION X TO ___.

A LISTING OF THE OBSERVATIONS IN SAMPLE NO. ___ FOLLOWS.*

1) x_1 x_2 ------- x_8

2) x_9 x_{10} ------- x_{16}

) ----------- x_i -----

) -----------------x_n

* The Chi-square tests for a given problem deck setup are numbered consecutively. The sequence of the tests is as follows: the INTER(I) tests on the 1st transformation of the first sample, the INTER(I) tests on the 1st transformation of the second sample, etc., the INTERP(I) tests on the pooled sample. This sequence is repeated for the second transformation, third transformation, etc. The number of observations in each test, the value of INTER(I), and the transformation identification (see Transformations, Section III.A.) is printed for each test.

(Identification of sample as given on Card Type 3 or Card Type 6.)

CHI SQUARE TEST ___ WITH ___ OBSERVATIONS AND ___ INTERVALS. TRANSFORMATION X TO ___.

THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST ___ OBSERVATIONS IN EACH SUBSET OF THE ___ INTERVALS.

MEAN $= \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}$... STANDARD DEVIATION $= \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} = s$

RANGE $= x_{max} - x_{min}$ MAXIMUM $= x_{max}$ MINIMUM $= x_{min}$ INTERVAL WIDTH $= \frac{x_{max} - x_{min}}{\text{INTER(I)}} = D$

UPPER BOUND	FREQUENCY	BAR CHART		CHI SQU CONTR	OBS FR	THEO FR
$x_{min} + 1D$	f'_1	I				
$x_{min} + 2D$	f'_2	I	(Number of X's represents the observed frequency in each interval.)	$\frac{(f_1-\hat{\varphi}_1)^2}{\hat{\varphi}_1}$	f_1	$\hat{\varphi}_1$
.	.	.		.	.	.
$x_{min} + j'D$	$f'_{j'}$	I		$\frac{(f_j-\hat{\varphi}_j)^2}{\hat{\varphi}_j}$	f_j	$\hat{\varphi}_j$
.	.	.		.	.	.
x_{min} + INTER(I)D	$f'_{\text{INTER(I)}}$	I		$\frac{(f_m-\hat{\varphi}_m)^2}{\hat{\varphi}_m}$	f_m	$\hat{\varphi}_m$

X REPRESENTS ___.___ OBSERVATIONS

CHISQUARE $= \sum_{j=1}^{m} \frac{(f_j - \hat{\varphi}_j)^2}{\hat{\varphi}_j}$ WITH m-3 DEGREES OF FREEDOM

V. EXAMPLE PROBLEM

A. Description of Example Problem

The example problem consists of two samples, one with 150 observations (sample No. 1) and the other with 125 observations (sample No. 2). Both samples are approximately log-normally distributed. (The observations of sample No. 1 are the antilogs of 150 numbers generated from an approximately normal distribution with mean = 2.50 and standard deviation = 0.30. The observations of sample No. 2 are the antilogs of 125 numbers generated from an approximately normal distribution with mean = 2.20 and standard deviation = 0.30.) Samples No. 1 and 2 should, therefore, appear to be from non-normal parent populations, but the logarithmically transformed observations of both samples should appear to have been sampled from normal parent populations.

In the example problem, the Chi-square test was performed on the original values (X to X transformation) and on the logarithms (X to ln X transformation) for both sample No. 1 and sample No. 2.

Two sets of intervals (20 and 25) were chosen for sample No. 1, i.e., two Chi-square tests were performed on the original values of sample No. 1 and two on the logarithms of the original values.

Only one set of intervals (15) was chosen for sample No. 2, amounting to two Chi-square tests, one on the original values of sample No. 2 and one on the logarithms.

The option for pooling the two samples was exercised, with only one set of intervals (40) being chosen for the pooled sample. Two Chi-square tests were performed, therefore, on the pooled sample, one on

the pooled original observations and one on the pooled logarithms of the original observations. Therefore, a total of 8 Chi-square tests were performed for the example problem. In each of the 8 tests the minimum number of expected observations in each interval was specified as 5.

This example problem was fabricated merely to illustrate the various features of the CHITRAN program, especially the transformation option, and is not intended as a guide to the choice of input parameters for the general problem.

B. Listing of Input Cards for the Example Problem

A listing of the input for the previously described example problem is given on the following two pages. Because two transformations (X to X and X to ln X) were specified, it was necessary to put Card Types 6 and 7 on tape.

LISTING OF INPUT CARDS FOR THE EXAMPLE PROBLEM

COLUMN NUMBER

CARD TYPE	5	10	15	20	25	30
1	(8F10.0)					
2	2	5	1	2	1	2
3	SAMPLE	NO.1	AND	SAMPLE	NO.2	POOLED
4		5	1	40		
5	150	5	2	20	25	
	125	5	1	15		

COLUMN NUMBER

CARD TYPE	10	20	30	40	50	60	70	80
6	SAMPLE NO. 1							
7	20.4	12.0	16.0	8.7	10.0	15.8	12.0	18.0
	10.4	17.5	9.7	23.0	13.6	8.5	22.6	10.4
	15.2	10.3	13.4	15.3	12.1	11.5	12.0	17.1
	10.4	22.4	12.2	17.5	7.0	6.5	9.0	8.6
	10.3	8.1	17.0	9.5	11.8	9.9	14.8	7.4
	8.2	20.6	12.6	24.5	10.5	12.1	10.7	6.5
	9.3	11.4	9.6	7.1	15.0	11.2	11.5	18.9
	12.1	10.4	8.0	8.7	9.1	9.0	16.5	9.0
	5.7	12.1	11.9	18.0	12.7	10.3	16.0	18.1
	9.6	17.1	13.0	14.5	10.8	13.3	20.3	6.5
	10.1	10.7	9.5	14.7	11.6	10.3	17.0	8.7
	11.7	12.7	10.8	15.9	23.1	16.9	18.2	12.2
	10.0	8.7	20.5	15.8	8.6	10.1	7.1	15.4
	9.3	8.3	8.8	12.4	10.1	16.6	8.9	14.7
	19.5	12.6	9.1	11.6	8.2	15.6	12.4	13.6
	13.7	13.4	11.5	16.2	17.2	24.7	11.9	15.0
	14.6	24.3	15.9	9.3	9.3	7.6	12.5	8.0
	13.2	22.8	17.9	18.2	12.7	11.0	12.8	14.1
	10.0	15.2	21.7	10.8	11.4	12.6		
6	SAMPLE NO. 2							
7	13.3	11.5	5.6	7.3	10.5	17.5	18.5	5.5
	15.8	10.0	10.9	9.4	8.8	10.2	6.9	7.1
	8.0	7.0	14.0	8.7	10.0	11.1	10.9	10.8
	14.4	8.9	9.9	7.9	14.1	13.0	10.7	8.5
	8.4	11.9	11.8	6.7	8.6	6.7	14.0	10.2
	6.5	9.8	10.9	7.4	9.2	7.0	13.5	5.7
	7.4	7.5	5.4	6.7	7.3	5.1	11.4	7.6
	10.8	14.5	11.0	8.4	8.2	6.7	7.0	7.3
	7.2	6.2	6.6	6.4	8.1	16.6	8.0	10.2
	8.9	10.3	11.1	12.8	6.3	14.8	9.3	7.6
	7.1	10.5	11.3	7.8	8.9	12.5	9.1	10.6
	8.6	8.2	12.3	9.2	17.1	5.9	15.9	5.8
	9.5	10.8	6.6	7.6	15.6	7.4	6.1	16.7
	12.8	9.7	11.3	7.3	8.8	8.1	15.1	7.8
	12.8	7.4	16.5	9.4	10.5	8.0	9.7	9.6
	12.6	7.3	5.4	10.3	7.5			

C. Test Results and Example Output

Test Results

The output from the example problem is given here in order to exhibit a sample of the program output and also to illustrate the the use of the $\hat{\chi}^2$ statistic in testing the hypothesis of normality.

The first Chi-square test performed in the example problem is performed on sample No. 1 (with 150 observations). The sample frequency distribution is divided into 20 intervals, i.e., INTER(1) = 20 on the first Card Type 5 of the input deck. The "transformation" used is X to X. The computed $\hat{\chi}^2$ statistic for this first test is 30.64 with 12 degrees of freedom. Because $\hat{\chi}^2$ (12 degrees of freedom) is approximately distributed as χ^2 (12 degrees of freedom), the upper tail of the tabled χ^2 distribution can be used as a critical (rejection) region for testing the hypothesis of normality. For any given significance level, α, where α is the probability of rejecting the hypothesis of normality when the hypothesis is true, the lower bound of the critical region is defined as that value of χ^2 (12 degrees of freedom) for which the probability of $\hat{\chi}^2$ (12 degrees of freedom) being greater than or equal to χ^2 (12 degrees of freedom) is equal to α. At the $\alpha = 0.01$ level of significance, for example, the tabled value of χ^2 (12 degrees of freedom) is 26.22. Testing at the 1% level of significance, therefore, the critical region is $\hat{\chi}^2 > 26.22$. Since the computed value of $\hat{\chi}^2$ (= 30.64) is within the critical region, the hypothesis of normality must be rejected at this level of significance. (Rejection of normality is to be expected since this sample of data is, as mentioned, approximately log normally distributed.)

As a second example of the application of the $\hat{\chi}^2$ statistic in testing the hypothesis of normality and as an illustration of the effect of the logarithmic transformation, consider the fifth Chi-square test printed in the output. This Chi-square test uses the same input parameters as the first test, but the fifth test is performed on the logarithms of sample No. 1, not on the original values. The fifth test resulted in a $\hat{\chi}^2$ value of 15.79 with 12 degrees of freedom. Testing at the α = .01 significance level, the critical region is, again, $\hat{\chi}^2 > 26.22$. Since $\hat{\chi}^2$ is not in the critical region, the hypothesis of normality cannot be rejected, for this sample, at the .01 level of significance. (Again, this conclusion is to be expected since the logarithms of the observations of an approximately log normal distribution should be approximately normally distributed.) It should be mentioned that although the number of degrees of freedom (12) calculated for the fifth test was the same as that calculated for the first test, the degrees of freedom of $\hat{\chi}^2$ for the transformed observations of a sample do not always agree with the degrees of freedom for the original observations.

The other six $\hat{\chi}^2$ statistics calculated for the example problem (and given in the example output) may be tested in a similar manner against tabled χ^2 values. The tabled χ^2 distribution is not included in this report since it can be found in many elementary statistical texts.

Example Output

SAMPLE NO. 1

CHI - SQUARE TEST 1 WITH 150 OBSERVATIONS AND 20 INTERVALS.TRANSFORMATION X TO X.

A LISTING OF THE OBSERVATIONS IN SAMPLE NO. 1 FOLLOWS.

1)	.20400000E+02	.12000000E+02	.16000000E+02	.87000000E+01	.10000000E+02	.15800000E+02	.12000000E+02	.18000000E+02
2)	.1[illegible]400000E+02	.17500000E+02	.97000000E+01	.23000000E+02	.13600000E+02	.85000000E+01	.22600000E+02	.10400000E+02
3)	.15200000E+02	.10300000E+02	.13400000E+02	.15300000E+02	.12100000E+02	.11500000E+02	.12000000E+02	.17100000E+02
4)	.11400000E+02	.22400000E+02	.12200000E+02	.17500000E+02	.70000000E+01	.65000000E+01	.90000000E+01	.86000000E+01
5)	.1[illegible]300000E+02	.31000000E+01	.17000000E+02	.95000000E+01	.11800000E+02	.99000000E+01	.14800000E+02	.74000000E+01
6)	.82000000E+01	.20600000E+02	.12600000E+02	.24500000E+02	.10500000E+02	.12100000E+02	.10700000E+02	.65000000E+01
7)	.93000000E+01	.11400000E+02	.96000000E+01	.71000000E+01	.15000000E+02	.11200000E+02	.11500000E+02	.18900000E+02
8)	.12100000E+02	.10400000E+02	.80000000E+01	.87000000E+01	.91000000E+01	.90000000E+01	.16500000E+02	.93000000E+01
9)	.57000000E+01	.12100000E+02	.11900000E+02	.18000000E+02	.12700000E+02	.10300000E+02	.16000000E+02	.13100000E+02
10)	.95000000E+01	.17100000E+02	.13000000E+02	.14500000E+02	.19800000E+02	.13300000E+02	.20300000E+02	.65000000E+01
11)	.10100000E+02	.10700000E+02	.95000000E+01	.14700000E+02	.11600000E+02	.10300000E+02	.17000000E+02	.87000000E+01
12)	.11700000E+02	.12700000E+02	.10800000E+02	.15900000E+02	.23100000E+02	.16900000E+02	.18200000E+02	.12200000E+02
13)	.1[illegible]E+02	.87000000E+01	.20500000E+02	.15800000E+02	.86000000E+01	.10100000E+02	.71000000E+01	.15400000E+02
14)	.83000000E+01	.83000000E+01	.88000000E+01	.12400000E+02	.10100000E+02	.16600000E+02	.89000000E+01	.14700000E+02
15)	.19500000E+02	.12600000E+02	.91000000E+01	.11600000E+02	.82000000E+01	.15600000E+02	.12400000E+02	.13500000E+02
16)	.13700000E+02	.13400000E+02	.11500000E+02	.16200000E+02	.17200000E+02	.24700000E+02	.11900000E+02	.15000000E+02
17)	.1[illegible]600000E+02	.24300000E+02	.15900000E+02	.93000000E+01	.93000000E+01	.76000000E+01	.12500000E+02	.80000000E+01
18)	.13200000E+02	.22800000E+02	.17900000E+02	.18200000E+02	.12700000E+02	.11000000E+02	.12800000E+02	.14100000E+02
19)	.10000000E+02	.15200000E+02	.21700000E+02	.19800000E+02	.11400000E+02	.12600000E+02		

SAMPLE N . 1

CHI SQUARE TEST 1 WITH 150 OBSERVATIONS AND 20 INTERVALS.TRANSFORMATION X TO X.
THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST 5 OBSERVATIONS IN EACH OF A SUBSET OF THE 20 INTERVALS.

MEAN= .12945333E+02 ...STANDARD DEVIATION= .42691745E+01

RANGE = .19000000E+02 MAXIMUM = .24700000E+02 MINIMUM = .57000000E+01 INTERVAL WIDTH = .95000000E+00

UPPER BOUND	FREQUENCY	BAR CHART	CHI SQU CONTR	OBS FR	THEO FR
6.6500	4.	IXXXX	4.045	4.0	10.524
7.6000	4.	IXXXX	0.305	4.0	5.267
8.5500	8.	IXXXXXXXX	0.158	8.0	6.951
9.5000	17.	IXXXXXXXXXXXXXXXXX	7.828	17.0	8.732
10.4500	20.	IXXXXXXXXXXXXXXXXXXXX	8.748	20.0	10.442
11.4000	10.	IXXXXXXXXXX	0.299	10.0	11.886
12.3500	18.	IXXXXXXXXXXXXXXXXXX	2.036	18.0	12.879
13.3000	12.	IXXXXXXXXXXXX	0.124	12.0	13.283
14.2500	7.	IXXXXXXX	2.799	7.0	13.041
15.2000	7.	IXXXXXXX	2.208	7.0	12.187
16.1500	11.	IXXXXXXXXXXX	0.002	11.0	10.841
17.1000	6.	IXXXXXX	1.101	6.0	9.180
18.0500	8.	IXXXXXXXX	0.049	8.0	7.399
19.0000	4.	IXXXX	0.495	4.0	5.677
19.9500	1.	IX			
20.9000	4.	IXXXX			
21.8500	1.	IX			
22.8000	3.	IXXX			
23.7500	2.	IXX			
24.7000	3.	IXXX	0.448	14.0	11.710

X REPRESENTS 1.0000 OBSERVATIONS.

CHISQUARE = 30.644881 WITH 12 DEGREES OF FREEDOM.

SAMPLE NO. 1

CHI SQUARE TEST 2 WITH 150 OBSERVATIONS AND 25 INTERVALS.TRANSFORMATION X TO X.
THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST 5 OBSERVATIONS IN EACH OF A SUBSET OF THE 25 INTERVALS.

MEAN= .12945333E+02 ...STANDARD DEVIATION= .42591745E+01

RANGE = .19000000E+02 MAXIMUM = .24700000E+02 MINIMUM = .57000000E+01 INTERVAL WIDTH = .76000000E+00

UPPER BOUND	FREQUENCY	BAR CHART	CHI SQU CONTR	OBS FR	THEO FR
6.4600	1.	IX	7.759	1.0	9.655
7.2200	6.	IXXXXXX			
7.9800	2.	IXX	0.057	8.0	8.705
8.7400	13.	IXXXXXXXXXXXXX	8.223	13.0	5.985
9.5000	11.	IXXXXXXXXXXX	2.102	11.0	7.129
10.2600	12.	IXXXXXXXXXXXX	1.730	12.0	8.227
11.0200	15.	IXXXXXXXXXXXXXXX	3.658	15.0	9.199
11.7800	9.	IXXXXXXXXX	0.094	9.0	9.966
12.5400	15.	IXXXXXXXXXXXXXXX	1.970	15.0	10.461
13.3000	9.	IXXXXXXXXX	0.252	9.0	10.639
14.0600	6.	IXXXXXX	1.917	6.0	10.483
14.8200	6.	IXXXXXX	1.605	6.0	10.008
15.5800	6.	IXXXXXX	1.146	6.0	9.257
16.3400	8.	IXXXXXXXX	0.011	8.0	8.297
17.1000	5.	IXXXXX	0.674	5.0	7.204
17.8600	5.	IXXXXX	0.186	5.0	6.061
18.6200	6.	IXXXXXX			
19.3800	1.	IX	0.384	7.0	8.843
20.1400	1.	IX			
20.9000	4.	IXXXX			
21.6600	0.	I			
22.4200	2.	IXX			
23.1800	4.	IXXXX			
23.9400	0.	I			
24.7000	3.	IXXX	1.717	14.0	9.881

X REPRESENTS 1.0000 OBSERVATIONS.

CHISQUARE = 33.484312 WITH 14 DEGREES OF FREEDOM.

SAMPLE NO. 2

CHI SQUARE TEST 3 WITH 125 OBSERVATIONS AND 15 INTERVALS.TRANSFORMATION X TO X.

A LISTING OF THE OBSERVATIONS IN SAMPLE NO. 2 FOLLOWS.

1)	.13300000E+02	.11500000E+02	.56000000E+01	.73000000E+01	.10500000E+02	.17500000E+02	.18500000E+02	.55000000E+01
2)	.15800000E+02	.10000000E+02	.10900000E+02	.94000000E+01	.88000000E+01	.10200000E+02	.69000000E+01	.71000000E+01
3)	.80000000E+01	.70000000E+01	.14000000E+02	.87000000E+01	.10000000E+02	.11100000E+02	.10900000E+02	.10300000E+02
4)	.14400000E+02	.89000000E+01	.99000000E+01	.79000000E+01	.14100000E+02	.13000000E+02	.10700000E+02	.85000000E+01
5)	.84000000E+01	.11900000E+02	.11800000E+02	.67000000E+01	.86000000E+01	.67000000E+01	.14000000E+02	.10200000E+02
6)	.65000000E+01	.98000000E+01	.10900000E+02	.74000000E+01	.92000000E+01	.70000000E+01	.13500000E+02	.57000000E+01
7)	.74000000E+01	.75000000E+01	.54000000E+01	.67000000E+01	.73000000E+01	.51000000E+01	.11400000E+02	.76000000E+01
8)	.10800000E+02	.14500000E+02	.11000000E+02	.84000000E+01	.82000000E+01	.67000000E+01	.70000000E+01	.73000000E+01
9)	.72000000E+01	.62000000E+01	.66000000E+01	.64000000E+01	.81000000E+01	.16600000E+02	.80000000E+01	.10200000E+02
10)	.89000000E+01	.10300000E+02	.11100000E+02	.12800000E+02	.63000000E+01	.14800000E+02	.93000000E+01	.76000000E+01
11)	.71000000E+01	.10500000E+02	.11300000E+02	.78000000E+01	.89000000E+01	.12500000E+02	.91000000E+01	.10600000E+02
12)	.96000000E+01	.82000000E+01	.12300000E+02	.92000000E+01	.17100000E+02	.59000000E+01	.15900000E+02	.58000000E+01
13)	.95000000E+01	.10800000E+02	.66000000E+01	.76000000E+01	.15600000E+02	.74000000E+01	.61000000E+01	.16700000E+02
14)	.12800000E+02	.97000000E+01	.11300000E+02	.73000000E+01	.88000000E+01	.81000000E+01	.15100000E+02	.78000000E+01
15)	.12800000E+02	.74000000E+01	.16500000E+02	.94000000E+01	.10500000E+02	.80000000E+01	.97000000E+01	.95000000E+01
16)	.12600000E+02	.73000000E+01	.54000000E+01	.10300000E+02	.75000000E+01			

```
SAMPLE NO. 2

CHI SQUARE TEST   3 WITH   125 OBSERVATIONS AND   15 INTERVALS.TRANSFORMATION X TO X.
THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST    5 OBSERVATIONS IN EACH OF A SUBSET OF THE   15 INTERVALS.

MEAN= .97336000E+01 ...STANDARD DEVIATION= .33426523E+01

RANGE  =  .13400000E+02   MAXIMUM =  .13500000E+02   MINIMUM =  .51000000E+01   INTERVAL WIDTH =  .89333333E+00

    UPPER BOUND  FREQUENCY     BAR CHART                                        CHI SQU CONTR    OBS FR     THEO FR
        5.9933       8.      IXXXXXXXX                                                2.362        8.0      13.686
        6.8867      11.      IXXXXXXXXXXX                                             0.993       11.0       6.155
        7.7800      21.      IXXXXXXXXXXXXXXXXXXXXX                                   9.882       21.0      10.712
        8.6733      15.      IXXXXXXXXXXXXXXX                                         0.336       15.0      12.916
        9.5667      13.      IXXXXXXXXXXXXX                                           0.118       13.0      14.297
       10.4600      12.      IXXXXXXXXXXXX                                            0.440       12.0      14.528
       11.3533      16.      IXXXXXXXXXXXXXXXX                                        0.443       16.0      13.551
       12.2467       4.      IXXXX                                                    4.982        4.0      11.603
       13.1400       7.      IXXXXXXX                                                 0.493        7.0       9.121
       14.0333       4.      IXXXX                                                    1.012        4.0       6.581
       14.9267       4.      IXXXX
       15.8200       3.      IXXX
       16.7133       4.      IXXXX
       17.6067       2.      IXX
       18.5000       1.      IX                                                       1.748       14.0       9.851

X REPRESENTS    1.0000 OBSERVATIONS.

CHISQUARE =        22.808300 WITH    8 DEGREES OF FREEDOM.
```

SAMPLE NO.1 AND SAMPLE NO.2 POOLED

CHI SQUARE TEST WITH 275. POOLED OBSERVATIONS AND 40 INTERVALS.TRANSFORMATION X TO X.
THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST 5 OBSERVATIONS IN EACH OF A SUBSET OF THE 40 INTERVALS.

MEAN= .00000000E 00 ...STANDARD DEVIATION= .37619702E+01

RANGE = .19000000E+02 MAXIMUM = .11754667E+02 MINIMUM = -.72453333E+01 INTERVAL WIDTH = .47500000E-00

UPPER BOUND	FREQUENCY	BAR CHART	CHI SQU CONTR	OBS FR	THEO FR
-6.7703	1.	IX	7.989	1.0	9.988
-6.2953	3.	IXXX			
-5.8203	3.	IXXX	0.109	6.0	6.863
-5.3453	1.	IX			
-4.8703	3.	IXXX	3.704	4.0	10.123
-4.3953	6.	IXXXXXX	0.037	6.0	6.492
-3.9203	17.	IXXXXXXXXXXXXXXXXX	11.943	17.0	7.522
-3.4453	9.	IXXXXXXXXX	0.021	9.0	8.578
-2.9703	15.	IXXXXXXXXXXXXXXX	2.997	15.0	9.629
-2.4953	21.	IXXXXXXXXXXXXXXXXXXXXX	10.098	21.0	10.636
-2.0203	20.	IXXXXXXXXXXXXXXXXXXXX	6.154	20.0	11.564
-1.5453	12.	IXXXXXXXXXXXX	0.011	12.0	12.374
-1.0703	14.	IXXXXXXXXXXXXXX	0.072	14.0	13.032
-.5953	18.	IXXXXXXXXXXXXXXXXXX	1.493	18.0	13.508
-.1203	17.	IXXXXXXXXXXXXXXXXX	0.752	17.0	13.781
0.3547	8.	IXXXXXXXX	2.462	8.0	13.836
0.8297	14.	IXXXXXXXXXXXXXX	0.008	14.0	13.673
1.3047	10.	IXXXXXXXXXX	0.818	10.0	13.298
1.7797	10.	IXXXXXXXXXX	0.585	10.0	12.729
2.2547	5.	IXXXXX	4.076	5.0	11.992
2.7297	6.	IXXXXXX	2.357	6.0	11.119
3.2047	11.	IXXXXXXXXXXX	0.072	11.0	10.147
3.6797	5.	IXXXXX	1.857	5.0	9.113
4.1547	4.	IXXXX	2.042	4.0	8.056
4.6297	9.	IXXXXXXXXX	0.140	6.0	7.009
5.1047	6.	IXXXXXX	0.000	6.0	6.001
5.5797	4.	IXXXX	0.221	4.0	5.057
6.0547	2.	IXX			
6.5297	2.	IXX	1.719	4.0	7.619
7.0047	4.	IXXXX			
7.4797	3.	IXXX			
7.9547	3.	IXXX			
8.4297	0.	I			
8.9047	2.	IXX			
9.3797	0.	I			
9.8547	3.	IXXX			
10.3297	2.	IXX			
10.8047	0.	I			
11.2797	0.	I			
11.7547	3.	IXXX	6.571	20.0	11.360

X REPRESENTS 1.0000 OBSERVATIONS.

CHISQUARE = 69.306985 WITH 22 DEGREES OF FREEDOM.

SAMPLE NO. 1

CHI SQUARE TEST 5 WITH 150 OBSERVATIONS AND 20 INTERVALS.TRANSFORMATION X TO LN X.

A LISTING OF THE OBSERVATIONS IN SAMPLE NO. 1 FOLLOWS.

1)	.[illegible]1553490E+01	.24849066E+01	.27725887E+01	.21633230E+01	.23025851E+01	.27600099E+01	.24849066E+01	.2[illegible]03718E+01
2)	.23418058E+01	.28622009E+01	.22721259E+01	.31354942E+01	.26100698E+01	.21400662E+01	.31179499E+01	.23418058E+01
3)	.27212954E+01	.23321439E+01	.25952547E+01	.27278528E+01	.24932055E+01	.24423470E+01	.24849066E+01	.23390785E+01
4)	.23418058E+01	.31090610E+01	.25014360E+01	.28622009E+01	.19459101E+01	.18718022E+01	.21972246E+01	.21517622E+01
5)	.23321439E+01	.20918641E+01	.28332133E+01	.22512918E+01	.24680995E+01	.22925348E+01	.26946272E+01	.20014800E+01
6)	.21041342E+01	.30252911E+01	.25336968E+01	.31986731E+01	.23513753E+01	.24932055E+01	.23702437E+01	.18718022E+01
7)	.22300144E+01	.24336134E+01	.22617631E+01	.19600948E+01	.27080502E+01	.24159138E+01	.24423470E+01	.2[illegible]391619E+01
8)	.24932055E+01	.23418058E+01	.20794415E+01	.21633230E+01	.22082744E+01	.21972246E+01	.28033604E+01	.21972246E+01
9)	.17404662E+01	.24932055E+01	.24765384E+01	.28903718E+01	.25416020E+01	.23321439E+01	.27725887E+01	.2[illegible]959119E+01
10)	.22617631E+01	.23390785E+01	.25649494E+01	.26741486E+01	.23795461E+01	.25877640E+01	.30106209E+01	.18718022E+01
11)	.23125354E+01	.23702437E+01	.22512918E+01	.26878475E+01	.24510051E+01	.23321439E+01	.28332133E+01	.21633230E+01
12)	.24595888E+01	.25416020E+01	.23795461E+01	.27663191E+01	.31398326E+01	.28273136E+01	.29014216E+01	.25014360E+01
13)	.23025851E+01	.21633230E+01	.30204249E+01	.27600099E+01	.21517622E+01	.23125354E+01	.19600948E+01	.27343675E+01
14)	.22300144E+01	.21162555E+01	.21747517E+01	.25176965E+01	.23125354E+01	.28094027E+01	.21860513E+01	.26878475E+01
15)	.29704145E+01	.25336968E+01	.22082744E+01	.24510051E+01	.21041342E+01	.27472709E+01	.25176965E+01	.26100698E+01
16)	.26173958E+01	.25952547E+01	.24423470E+01	.27850112E+01	.28449094E+01	.32068032E+01	.24765384E+01	.27080502E+01
17)	.26810215E+01	.31904764E+01	.27663191E+01	.22300144E+01	.22300144E+01	.20281482E+01	.25257286E+01	.20794415E+01
18)	.25802168E+01	.31267605E+01	.28848007E+01	.29014216E+01	.25416020E+01	.23978953E+01	.25494452E+01	.26461748E+01
19)	.23025851E+01	.27212954E+01	.30773123E+01	.23795461E+01	.24336134E+01	.25336968E+01		

SAMPLE NO. 1

CHI SQUARE TEST 5 WITH 150 OBSERVATIONS AND 20 INTERVALS.TRANSFORMATION X TO LN X.
THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST 5 OBSERVATIONS IN EACH OF A SUBSET OF THE 20 INTERVALS.

MEAN= .25034583E+01 ...STANDARD DEVIATION= .31952035E-00

RANGE = .14663371E+01 MAXIMUM = .32068032E+01 MINIMUM = .17404662E+01 INTERVAL WIDTH = .73316853E-01

UPPER BOUND	FREQUENCY	BAR CHART	CHI SQU CONTR	OBS FR	THEO FR
1.8138	1.	IX			
1.8871	3.	IXXX			
1.9604	3.	IXXX	0.050	7.0	6.430
2.0337	2.	IXX			
2.1071	5.	IXXXXX	0.510	7.0	9.161
2.1804	9.	IXXXXXXXXX	0.487	9.0	7.136
2.2537	12.	IXXXXXXXXXXXX	0.977	12.0	9.029
2.3270	10.	IXXXXXXXXXX	0.065	10.0	10.841
2.4003	15.	IXXXXXXXXXXXXXXX	0.568	15.0	12.352
2.4736	10.	IXXXXXXXXXX	0.843	10.0	13.355
2.5470	20.	IXXXXXXXXXXXXXXXXXXXX	2.896	20.0	13.701
2.6203	9.	IXXXXXXXXX	1.411	9.0	13.339
2.6936	5.	IXXXXX	4.351	5.0	12.322
2.7669	12.	IXXXXXXXXXXXX	0.133	12.0	10.902
2.8402	10.	IXXXXXXXXXX	0.114	10.0	8.986
2.9135	9.	IXXXXXXXXX	0.512	9.0	7.094
2.9869	2.	IXX	2.067	2.0	5.314
3.0602	4.	IXXXX			
3.1335	4.	IXXXX			
3.2068	5.	IXXXXX	0.809	13.0	10.137

X REPRESENTS 1.0000 OBSERVATIONS.

CHISQUARE = 15.793015 WITH 12 DEGREES OF FREEDOM.

SAMPLE NO. 1

CHI SQUARE TEST 6 WITH 150 OBSERVATIONS AND 25 INTERVALS.TRANSFORMATION X TO LN X.
THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST 5 OBSERVATIONS IN EACH OF A SUBSET OF THE 25 INTERVALS.

MEAN= .25094583E+01 ...STANDARD DEVIATION= .31952035E-00

RANGE = .14663371E+01 MAXIMUM = .32068032E+01 MINIMUM = .17404662E+01 INTERVAL WIDTH = .58653483E-01

UPPER BOUND	FREQUENCY	BAR CHART	CHI SQU CONTR	OBS FR	THEO FR
1.7991	1.	IX			
1.8578	0.	I			
1.9164	3.	IXXX			
1.9751	3.	IXXX	0.001	7.0	7.083
2.0337	2.	IXX			
2.0924	3.	IXXX	0.725	5.0	7.301
2.1510	4.	IXXXX	0.303	4.0	5.264
2.2097	13.	IXXXXXXXXXXXXX	6.610	13.0	6.464
2.2683	8.	IXXXXXXXX	0.014	8.0	7.675
2.3270	8.	IXXXXXXXX	0.075	8.0	8.812
2.3857	14.	IXXXXXXXXXXXXXX	1.818	14.0	9.782
2.4443	7.	IXXXXXXX	1.168	7.0	10.502
2.5030	15.	IXXXXXXXXXXXXXXX	1.541	15.0	10.901
2.5616	10.	IXXXXXXXXXX	0.081	10.0	10.942
2.6203	8.	IXXXXXXXX	0.646	8.0	10.619
2.6789	2.	IXX	6.368	2.0	9.966
2.7376	10.	IXXXXXXXXXX	0.101	10.0	9.044
2.7962	8.	IXXXXXXXX	0.001	8.0	7.936
2.8549	8.	IXXXXXXXX	0.238	8.0	6.734
2.9135	8.	IXXXXXXXX	1.109	8.0	5.525
2.9722	2.	IXX			
3.0308	4.	IXXXX	0.393	6.0	7.746
3.0895	1.	IX			
3.1481	5.	IXXXXX			
3.2068	3.	IXXX	0.218	9.0	7.705

X REPRESENTS 1.0000 OBSERVATIONS.

CHISQUARE = 21.409583 WITH 15 DEGREES OF FREEDOM.

SAMPLE NO. 2

CHI SQUARE TEST 7 WITH 125 OBSERVATIONS AND 15 INTERVALS.TRANSFORMATION X TO LN X.

A LISTING OF THE OBSERVATIONS IN SAMPLE NO. 2 FOLLOWS.

1)	.25877640E+01	.24423470E+01	.17227666E+01	.19878743E+01	.23513753E+01	.28622009E+01	.29177707E+01	.17047481E+01
2)	.27600099E+01	.23025851E+01	.23887628E+01	.22407097E+01	.21747517E+01	.23223877E+01	.19315214E+01	.19600948E+01
3)	.20794415E+01	.19459101E+01	.26390573E+01	.21633230E+01	.23025851E+01	.24069451E+01	.23887628E+01	.23795461E+01
4)	.26672282E+01	.21860513E+01	.22925348E+01	.20668628E+01	.26461748E+01	.25649494E+01	.23702437E+01	.21400662E+01
5)	.21282317E+01	.24765384E+01	.24680995E+01	.19021075E+01	.21517622E+01	.19021075E+01	.26390573E+01	.23223877E+01
6)	.18718022E+01	.22823824E+01	.23887628E+01	.20014800E+01	.22192035E+01	.19459101E+01	.26026897E+01	.17404662E+01
7)	.20014800E+01	.20149030E+01	.16863990E+01	.19021075E+01	.19878743E+01	.16292405E+01	.24336134E+01	.20281482E+01
8)	.23795461E+01	.26741486E+01	.23978953E+01	.21282317E+01	.21041342E+01	.19021075E+01	.19459101E+01	.19878743E+01
9)	.19740810E+01	.18245493E+01	.18870696E+01	.18562980E+01	.20918641E+01	.28094027E+01	.20794415E+01	.23223877E+01
10)	.21860513E+01	.23321439E+01	.24069451E+01	.25494452E+01	.18405496E+01	.26946272E+01	.22300144E+01	.20281482E+01
11)	.19600948E+01	.23513753E+01	.24248027E+01	.20541237E+01	.21860513E+01	.25257286E+01	.22082744E+01	.23608540E+01
12)	.21517622E+01	.21041342E+01	.25095993E+01	.22192035E+01	.28390785E+01	.17749524E+01	.27663191E+01	.17578579E+01
13)	.22512918E+01	.23795461E+01	.18870696E+01	.20281482E+01	.27472709E+01	.20014800E+01	.18082888E+01	.28154087E+01
14)	.25494452E+01	.22721259E+01	.24248027E+01	.19878743E+01	.21747517E+01	.20918641E+01	.27146947E+01	.20541237E+01
15)	.25494452E+01	.20014800E+01	.28033604E+01	.22407097E+01	.23513753E+01	.20794415E+01	.22721259E+01	.22617631E+01
16)	.25336968E+01	.19878743E+01	.16863990E+01	.23321439E+01	.20149030E+01			

SAMPLE NO. 7

CHI SQUARE TEST 7 WITH 125 OBSERVATIONS AND 15 INTERVALS.TRANSFORMATION X TO LN X.
THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST 5 OBSERVATIONS IN EACH OF A SUBSET OF THE 15 INTERVALS.

MEAN .2[illegible]E+01 ...STANDARD DEVIATION= .30007938E-00

RANGE .12885302E+01 MAXIMUM = .29177707E+01 MINIMUM = .16292405E+01 INTERVAL WIDTH = .859020 13E-01

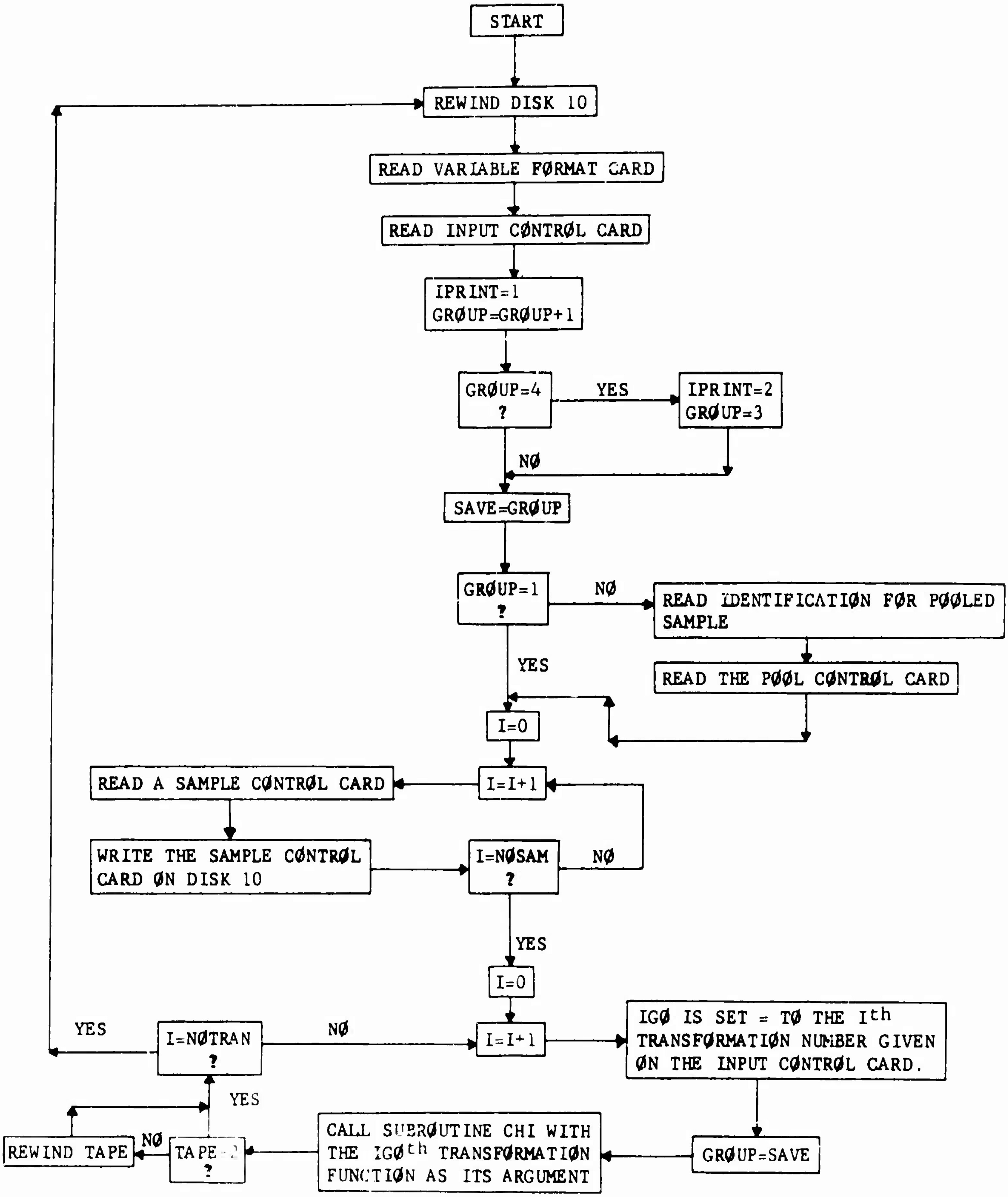
UPPER BOUND	FREQUENCY	BAR CHART	CHI SQU CONTR	OBS FR	THEO FR
1.7151	4.	IXXXX	0.357	4.0	5.387
1.8010	4.	IXXXX			
1.8869	5.	IXXXXX	0.193	9.0	10.420
1.9728	12.	IXXXXXXXXXXXX	1.291	12.0	8.657
2.0588	17.	IXXXXXXXXXXXXXXXXX	3.210	17.0	11.045
2.1447	11.	IXXXXXXXXXXX	0.305	11.0	12.991
2.2306	12.	IXXXXXXXXXXXX	0.309	12.0	14.086
2.3165	10.	IXXXXXXXXXX	1.182	10.0	14.079
2.4024	17.	IXXXXXXXXXXXXXXXXX	1.251	17.0	12.972
2.4883	8.	IXXXXXXXX	0.827	8.0	11.018
2.5742	7.	IXXXXXXX	0.307	7.0	8.627
2.6601	5.	IXXXXX	0.242	5.0	6.227
2.7460	4.	IXXXX			
2.8319	6.	IXXXXXX			
2.9178	3.	IXXX	1.298	13.0	9.490

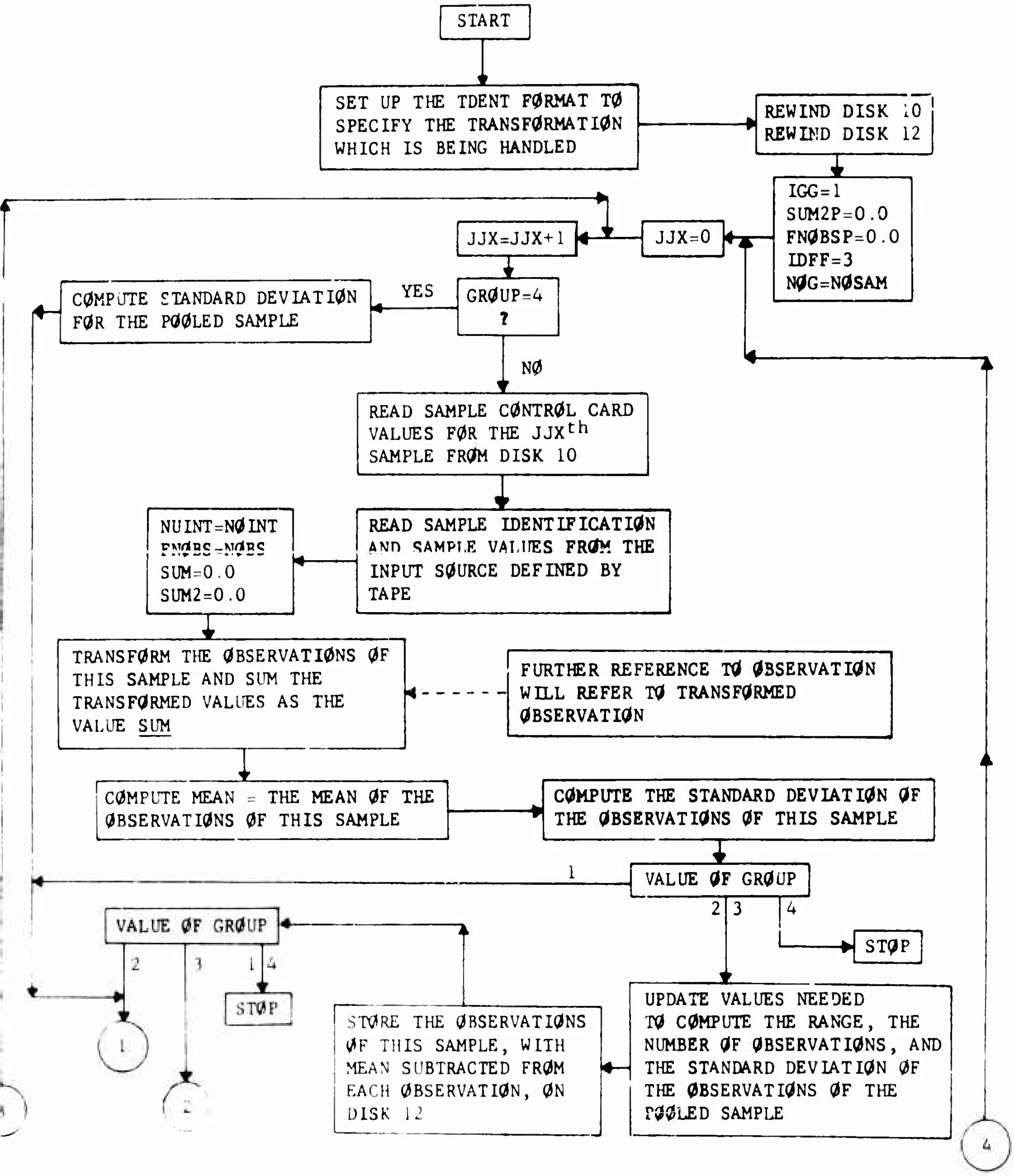
X REPRESENTS 1.0000 OBSERVATIONS.

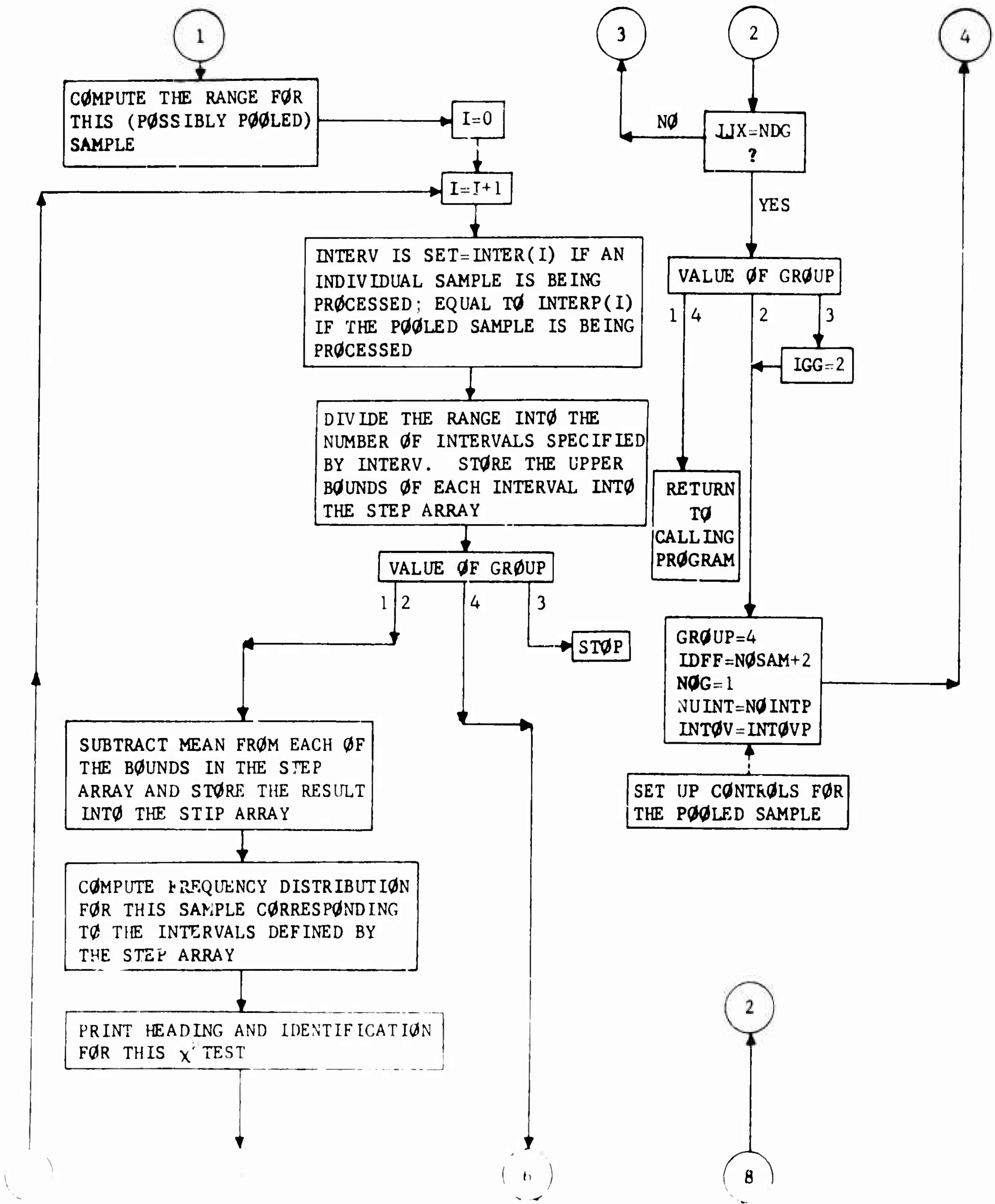
CHISQUARE = 10.771828 WITH 9 DEGREES OF FREEDOM.

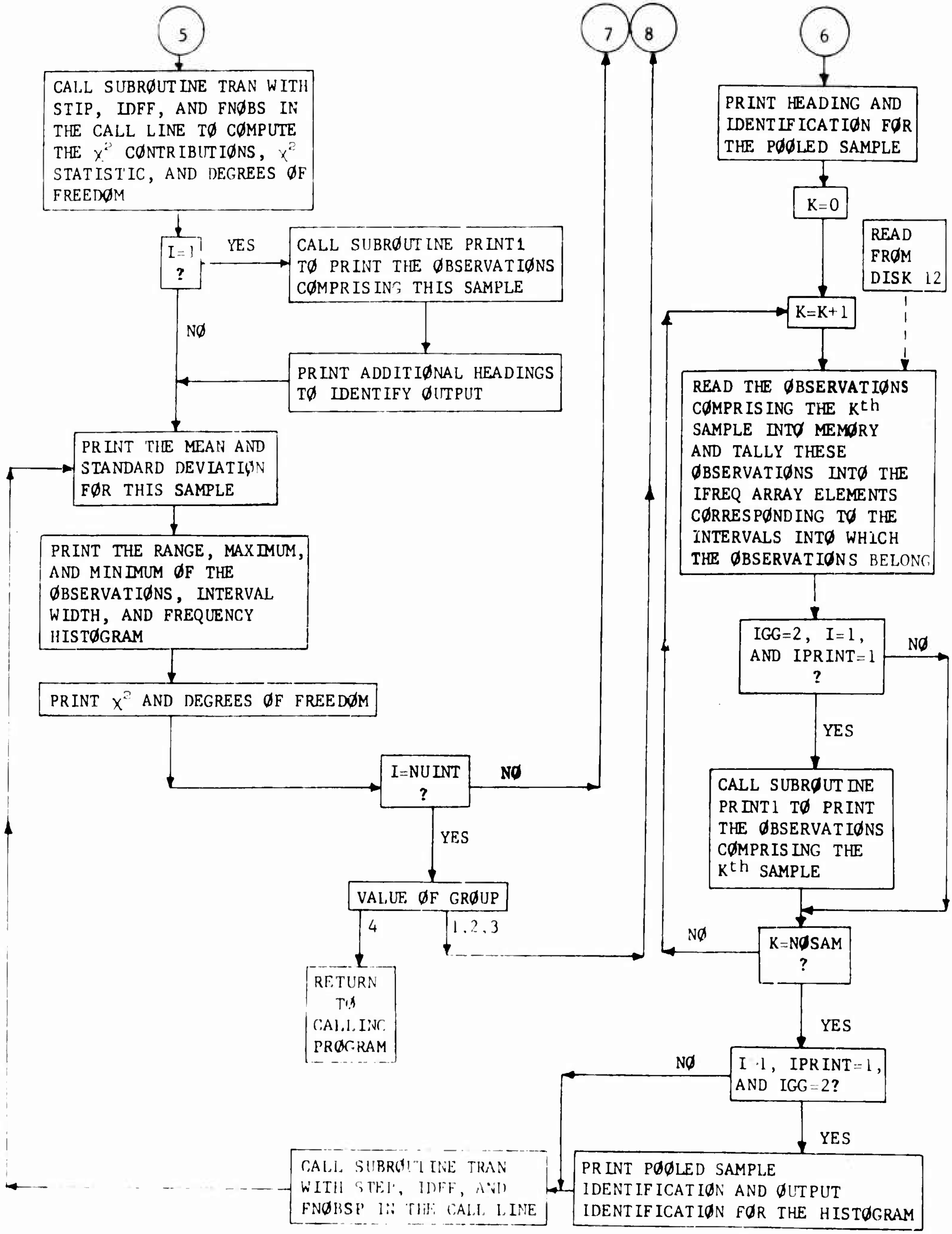
```
SAMPLE NO.1 AND SAMPLE NO.2 POOLED

CHI SQUARE TEST   8 WITH    275. POOLED OBSERVATIONS AND   40 INTERVALS.TRANSFORMATION X TO LN X.
THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT LEAST    5 OBSERVATIONS IN EACH OF A SUBSET OF THE   40 INTERVALS.

MEAN= .00000000E 00 ...STANDARD DEVIATION= .31084077E-00

RANGE =  .14663371E+01   MAXIMUM =  .69734499E+00   MINIMUM = -.76899217E+00   INTERVAL WIDTH =  .36658427E-01

 UPPER BOUND  FREQUENCY    BAR CHART                        CHI SQU CONTR   OBS FR    THEO FR
   -.7323        1.     IX
   -.6957        0.     I
   -.6590        0.     I
   -.6224        3.     IXXX                                    0.795       4.0       6.224
   -.5857        1.     IX
   -.5490        3.     IXXX
   -.5124        3.     IXXX                                    0.024       7.0       7.426
   -.4757        4.     IXXXX
   -.4391        2.     IXX                                     0.521       6.0       8.047
   -.4024        7.     IXXXXXXX                                0.640       7.0       5.179
   -.3657        4.     IXXXX                                   0.685       4.0       6.032
   -.3291       10.     IXXXXXXXXXX                             1.361      10.0       6.929
   -.2924       11.     IXXXXXXXXXXX                            1.265      11.0       7.849
   -.2558       12.     IXXXXXXXXXXXX                           1.190      12.0       8.769
   -.2191       12.     IXXXXXXXXXXXX                           0.566      12.0       9.662
   -.1825       12.     IXXXXXXXXXXXX                           0.215      12.0      10.498
   -.1458       15.     IXXXXXXXXXXXXXXX                        1.250      15.0      11.250
   -.1091       10.     IXXXXXXXXXX                             0.300      10.0      11.889
   -.0725        8.     IXXXXXXXX                               1.5[illegible]       8.0      12.391
   -.0358       13.     IXXXXXXXXXXXXX                          0.005      13.0      12.736
   0.0008       15.     IXXXXXXXXXXXXXXX                        0.338      15.0      12.910
   0.0375       13.     IXXXXXXXXXXXXX                          0.001      13.0      12.906
   0.0742        9.     IXXXXXXXXX                              1.090       9.0      12.724
   0.1108       11.     IXXXXXXXXXXX                            0.152      11.0      12.372
   0.1475        6.     IXXXXXX                                 2.897       6.0      11.863
   0.1841       13.     IXXXXXXXXXXXXX                          0.283      13.0      11.218
   0.2208       10.     IXXXXXXXXXX                             0.020      10.0      10.462
   0.2574        8.     IXXXXXXXX                               0.273       8.0       9.622
   0.2941        5.     IXXXXX                                  1.592       5.0       8.728
   0.3308       11.     IXXXXXXXXXXX                            1.306      11.0       7.807
   0.3674        5.     IXXXXX                                  0.517       5.0       6.888
   0.4041        7.     IXXXXXXX                                0.169       7.0       5.992
   0.4407        5.     IXXXXX                                  0.004       5.0       5.142
   0.4774        3.     IXXX
   0.5141        4.     IXXXX                                   0.121       7.0       7.331
   0.5507        4.     IXXXX
   0.5874        4.     IXXXX                                   1.236       6.0       5.413
   0.6240        4.     IXXXX
   0.6607        3.     IXXX
   0.6973        4.     IXXXX                                   1.050      11.0       8.087

X REPRESENTS    1.0000 OBSERVATIONS.

CHISQUARE =        21.423143 WITH  26 DEGREES OF FREEDOM.
```

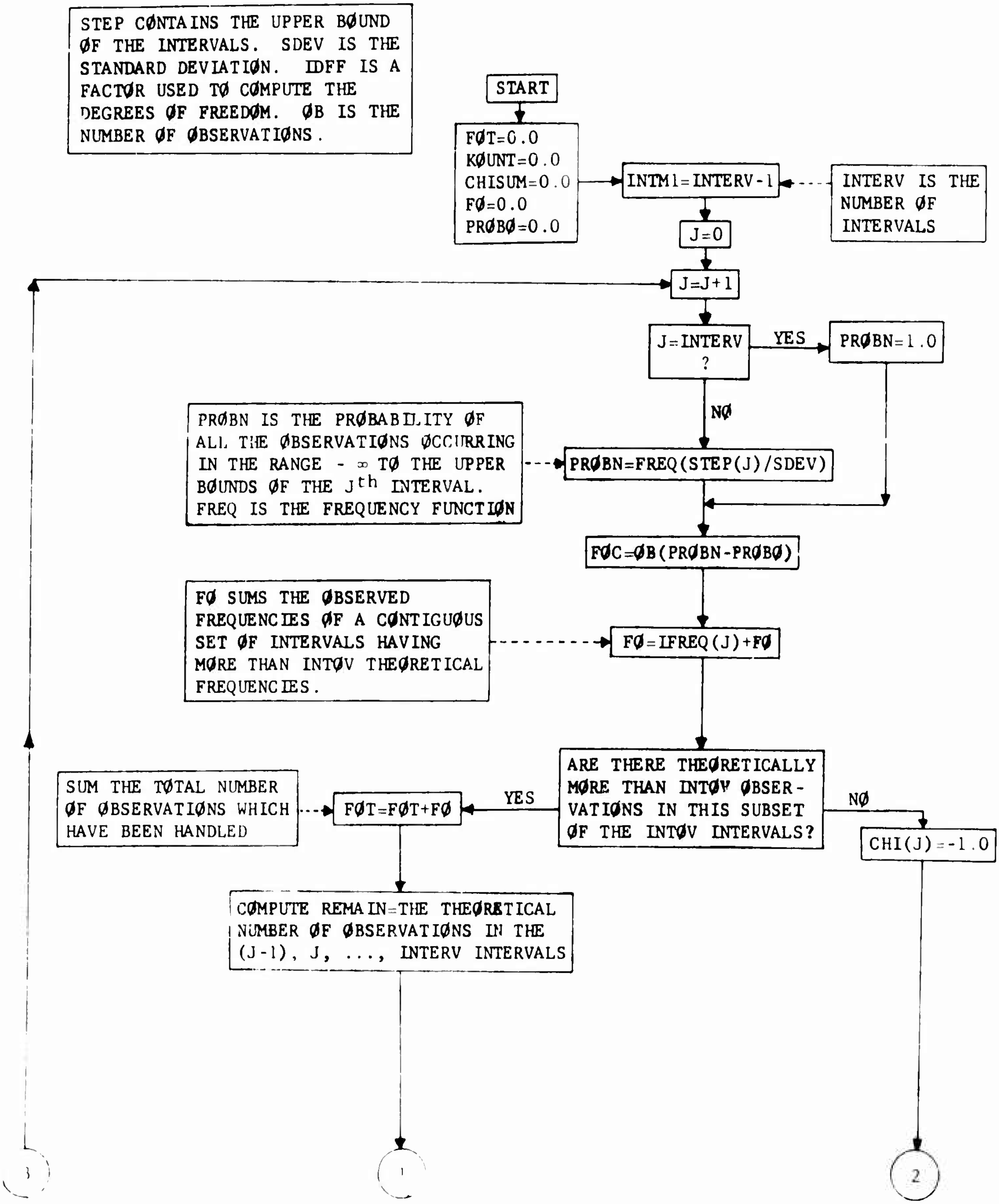
VI. FLOW CHARTS

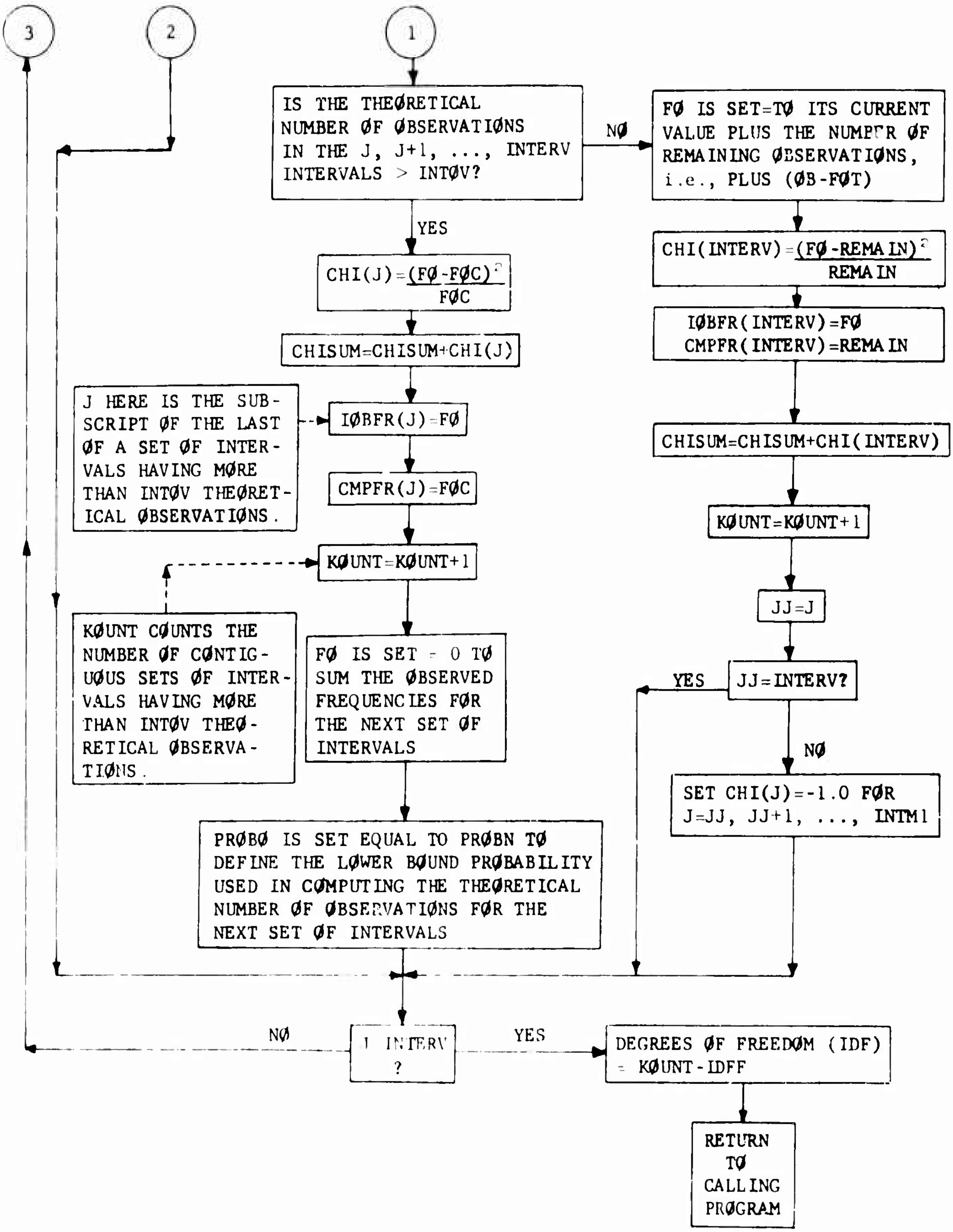



START
SET UP THE TDENT FØRMAT TØ SPECIFY THE TRANSFØRMATIØN WHICH IS BEING HANDLED
REWIND DISK 10
REWIND DISK 12
IGG=1
SUM2P=0.0
FNØBSP=0.0
IDFF=3
NØG=NØSAM
JJX=0
JJX=JJX+1
GRØUP=4 ?
YES
NØ
CØMPUTE STANDARD DEVIATIØN FØR THE PØØLED SAMPLE
READ SAMPLE CØNTRØL CARD VALUES FØR THE JJXth SAMPLE FRØM DISK 10
READ SAMPLE IDENTIFICATIØN AND SAMPLE VALUES FRØM THE INPUT SØURCE DEFINED BY TAPE
NUINT=NØINT
FNØBS=NØBS
SUM=0.0
SUM2=0.0
TRANSFØRM THE ØBSERVATIØNS ØF THIS SAMPLE AND SUM THE TRANSFØRMED VALUES AS THE VALUE SUM
FURTHER REFERENCE TØ ØBSERVATIØN WILL REFER TØ TRANSFØRMED ØBSERVATIØN
CØMPUTE MEAN = THE MEAN ØF THE ØBSERVATIØNS ØF THIS SAMPLE
CØMPUTE THE STANDARD DEVIATIØN ØF THE ØBSERVATIØNS ØF THIS SAMPLE
VALUE ØF GRØUP
1
2
3
4
STØP
UPDATE VALUES NEEDED TØ CØMPUTE THE RANGE, THE NUMBER ØF ØBSERVATIØNS, AND THE STANDARD DEVIATIØN ØF THE ØBSERVATIØNS ØF THE PØØLED SAMPLE
STØRE THE ØBSERVATIØNS ØF THIS SAMPLE, WITH MEAN SUBTRACTED FRØM EACH ØBSERVATIØN, ØN DISK 12
VALUE ØF GRØUP
2
3
1
4
STØP
1
2
4


1
COMPUTE THE RANGE FØR THIS (PØSSIBLY PØØLED) SAMPLE
I=0
I=I+1
INTERV IS SET=INTER(I) IF AN INDIVIDUAL SAMPLE IS BEING PRØCESSED; EQUAL TØ INTERP(I) IF THE PØØLED SAMPLE IS BEING PRØCESSED
DIVIDE THE RANGE INTØ THE NUMBER ØF INTERVALS SPECIFIED BY INTERV. STØRE THE UPPER BØUNDS ØF EACH INTERVAL INTØ THE STEP ARRAY
VALUE ØF GRØUP
1
2
4
3
STØP
SUBTRACT MEAN FRØM EACH ØF THE BØUNDS IN THE STEP ARRAY AND STØRE THE RESULT INTØ THE STIP ARRAY
CØMPUTE FREQUENCY DISTRIBUTIØN FØR THIS SAMPLE CØRRESPØNDING TØ THE INTERVALS DEFINED BY THE STEP ARRAY
PRINT HEADING AND IDENTIFICATIØN FØR THIS χ² TEST
3
2
4
NØ
IJX=NDG ?
YES
VALUE ØF GRØUP
1
4
2
3
IGG=2
RETURN TØ CALLING PRØGRAM
GRØUP=4
IDFF=NØSAM+2
NØG=1
NUINT=NØINTP
INTØV=INTØVP
SET UP CØNTRØLS FØR THE PØØLED SAMPLE
2
8


5
CALL SUBRØUTINE TRAN WITH STIP, IDFF, AND FNØBS IN THE CALL LINE TØ COMPUTE THE χ^2 CØNTRIBUTIØNS, χ^2 STATISTIC, AND DEGREES ØF FREEDØM
I=1 ?
YES
CALL SUBRØUTINE PRINT1 TØ PRINT THE ØBSERVATIØNS CØMPRISING THIS SAMPLE
NØ
PRINT ADDITIØNAL HEADINGS TØ IDENTIFY ØUTPUT
PRINT THE MEAN AND STANDARD DEVIATIØN FØR THIS SAMPLE
PRINT THE RANGE, MAXIMUM, AND MINIMUM ØF THE ØBSERVATIØNS, INTERVAL WIDTH, AND FREQUENCY HISTØGRAM
PRINT χ^2 AND DEGREES ØF FREEDØM
I=NUINT ?
NØ
YES
VALUE ØF GRØUP
4
1,2,3
RETURN TØ CALLING PRØGRAM
7
8
6
PRINT HEADING AND IDENTIFICATIØN FØR THE PØØLED SAMPLE
K=0
READ FRØM DISK 12
K=K+1
READ THE ØBSERVATIØNS CØMPRISING THE K^{th} SAMPLE INTØ MEMØRY AND TALLY THESE ØBSERVATIØNS INTØ THE IFREQ ARRAY ELEMENTS CØRRESPØNDING TØ THE INTERVALS INTØ WHICH THE ØBSERVATIØNS BELONG
IGG=2, I=1, AND IPRINT=1 ?
NØ
YES
CALL SUBRØUTINE PRINT1 TØ PRINT THE ØBSERVATIØNS CØMPRISING THE K^{th} SAMPLE
NØ
K=NØSAM ?
YES
NØ
I=1, IPRINT=1, AND IGG=2?
YES
PRINT PØØLED SAMPLE IDENTIFICATIØN AND ØUTPUT IDENTIFICATIØN FØR THE HISTØGRAM
CALL SUBRØUTINE TRAN WITH STEP, IDFF, AND FNØBSP IN THE CALL LINE


STEP CØNTAINS THE UPPER BØUND ØF THE INTERVALS. SDEV IS THE STANDARD DEVIATIØN. IDFF IS A FACTØR USED TØ CØMPUTE THE DEGREES ØF FREEDØM. ØB IS THE NUMBER ØF ØBSERVATIØNS.
START
FØT=0.0
KØUNT=0.0
CHISUM=0.0
FØ=0.0
PRØBØ=0.0
INTM1=INTERV-1
INTERV IS THE NUMBER ØF INTERVALS
J=0
J=J+1
J=INTERV ?
YES
PRØBN=1.0
NØ
PRØBN IS THE PRØBABILITY ØF ALL THE ØBSERVATIØNS ØCCURRING IN THE RANGE - ∞ TØ THE UPPER BØUNDS ØF THE Jth INTERVAL. FREQ IS THE FREQUENCY FUNCTIØN
PRØBN=FREQ(STEP(J)/SDEV)
FØC=ØB(PRØBN-PRØBØ)
FØ SUMS THE ØBSERVED FREQUENCIES ØF A CØNTIGUØUS SET ØF INTERVALS HAVING MØRE THAN INTØV THEØRETICAL FREQUENCIES.
FØ=IFREQ(J)+FØ
ARE THERE THEØRETICALLY MØRE THAN INTØV ØBSERVATIØNS IN THIS SUBSET ØF THE INTØV INTERVALS?
YES
NØ
SUM THE TØTAL NUMBER ØF ØBSERVATIØNS WHICH HAVE BEEN HANDLED
FØT=FØT+FØ
CHI(J)=-1.0
CØMPUTE REMAIN=THE THEØRETICAL NUMBER ØF ØBSERVATIØNS IN THE (J-1), J, ..., INTERV INTERVALS
3
1
2



3
2
1
IS THE THEØRETICAL NUMBER ØF ØBSERVATIØNS IN THE J, J+1, ..., INTERV INTERVALS > INTØV?
NØ
FØ IS SET=TØ ITS CURRENT VALUE PLUS THE NUMBER ØF REMAINING ØBSERVATIØNS, i.e., PLUS (ØB-FØT)
YES
CHI(J)=(FØ-FØC)²/FØC
CHI(INTERV)=(FØ-REMAIN)²/REMAIN
IØBFR(INTERV)=FØ
CMPFR(INTERV)=REMAIN
CHISUM=CHISUM+CHI(J)
J HERE IS THE SUBSCRIPT ØF THE LAST ØF A SET ØF INTERVALS HAVING MØRE THAN INTØV THEØRETICAL ØBSERVATIØNS.
IØBFR(J)=FØ
CHISUM=CHISUM+CHI(INTERV)
CMPFR(J)=FØC
KØUNT=KØUNT+1
KØUNT=KØUNT+1
JJ=J
KØUNT CØUNTS THE NUMBER ØF CØNTIGUØUS SETS ØF INTERVALS HAVING MØRE THAN INTØV THEØRETICAL ØBSERVATIØNS.
FØ IS SET = 0 TØ SUM THE ØBSERVED FREQUENCIES FØR THE NEXT SET ØF INTERVALS
YES
JJ=INTERV?
NØ
SET CHI(J)=-1.0 FØR J=JJ, JJ+1, ..., INTM1
PRØBØ IS SET EQUAL TO PRØBN TØ DEFINE THE LØWER BØUND PRØBABILITY USED IN CØMPUTING THE THEØRETICAL NUMBER ØF ØBSERVATIØNS FØR THE NEXT SET ØF INTERVALS
NØ
INTERV ?
YES
DEGREES ØF FREEDØM (IDF) = KØUNT-IDFF
RETURN TØ CALLING PRØGRAM

VII. PROGRAM LISTING

```
T          SUBTYPE,FIOD
B    20    IOD,$READER
B    30    IOD,$PRINTER
B    50    IOD,TAPE,,,,EVEN,,SAVE
B          REEL,PUL
B   128    IOD,DISK,,,2899
B   108    IOD,DISK,,,101
           END
T          SUBTYPE,FORTRAN,LMAP,PBIN
C   CHITRAN-CHI SQUARE TEST WITH TRANSFORMATIONS.
      COMMON MAX,MIN,OBSERV(14000),NOBS,STEP(400),IFREQ(401),SDEV,IDF
      COMMON CHS(400),CHISUM,IOBFR(400),CMPFR(400)
      COMMON INTOV,FNOBS,INTER(100),FGRAPH(65),STEP(400),TAPE,IGO
      COMMON NOJOBS,FORM(10),POOLID(10),OBSERP(14000),INTERP(100)
      COMMON NOBSA(500),INTOVP,NOINTP,GROUP,INTERV,IPRINT,Q
      DIMENSION IT(12)
      EQUIVALENCE (NOSAM,NOJOBS)
      INTEGER TAPE,GROUP,SAVE,Q
      EXTERNAL T1,T2,T3,T4,T5,T6,T7,T8,T9,T10,T11
   99 REWIND 10
C READ VARIABLE FORMAT CARD,THIS FORMAT IS USED TO READ THE OBSERVATIONS
      READ 400,FORM
  400 FORMAT(10A8)
      Q=0
      READ 100,NOSAM ,TAPE,GROUP,NOTRAN,(IT(I),I=1,NOTRAN)
  100 FORMAT(16I5)
      IF(GROUP.GT.3)STOP
      IPRINT=1
      GROUP=GROUP+1
      IF(GROUP.GT.1.AND.NOSAM.EQ.1)GROUP=1
      IF(GROUP.LE.3)GO TO 98
      IPRINT=2
      GROUP=3
   98 SAVE=GROUP
      IF(GROUP.EQ.1)GO TO 201
C READ ID FOR THE POOLED DATA.
      READ 400,POOLID
C READ POOL CONTROL CARD.
      READ 101,INTOVP,NOINTP,(INTERP(L),L=1,NOINTP)
  101 FORMAT(5X,15I5/((16I5)))
  201 DO 200 I=1,NOJOBS
C READ CONTROL CARDS FOR EACH SAMPLE.
      READ 100,NOBS,INTOV,NOINT,(INTER(L),L=1,NOINT)
      NOBSA(I)=NOBS
  200 WRITE(10)NOBS,INTOV,NOINT,(INTER (K),K=1,NOINT)
      DO 300 I=1,NOTRAN
      IGO=IT(I)
      GROUP=SAVE
      GO TO(1,2,3,4,5,6,7,8,9,10,11),IGO
    1 CALL CHI(T1)
      GO TO 250
    2 CALL CHI(T2)
      GO TO 250
    3 CALL CHI(T3)
      GO TO 250
    4 CALL CHI(T4)
      GO TO 250
    5 CALL CHI(T5)
      GO TO 250
    6 CALL CHI(T6)
```

```
      GO TO 250
    7 CALL CHI(T7)
      GO TO 250
    8 CALL CHI(T8)
      GO TO 250
    9 CALL CHI(T9)
      GO TO 250
   10 CALL  CHI(T10)
      GO TO 250
   11 CALL CHI(T11)
  250 IF(TAPE-2)251,300,251
  251 REWIND TAPE
  300 CONTINUE
      GO TO 99
      END
T        SUBTYPE,FORTRAN,LMAP,PBIN
      SUBROUTINE CHI(TFUNC)
      COMMON MAX,MIN,OBSERV(14000),NOBS,STIP(400),IFREQ(401),SDEV,IDF
      COMMON CHS(400),CHISUM,IOBFR(400),CMPFR(400)
      COMMON INTOV,FNOBS,INTER(100),FGRAPH(65),STEP(400),TAPE,IGO
      COMMON NOJOBS,FORM(10),POOLID(10),OBSERP(14000),INTERP(100)
      COMMON NOBSA(500),INTOVP,NOINTP,GROUP,INTERV,IPRINT,Q
      DIMENSION TDENT(6),TADD(11),TADD2(11)
      DIMENSION IDENT(10)
      EQUIVALENCE (BLANK,TADD2(1)),(XXX,TADD(1))
      REAL MEAN,MAX,MIN,MAXP,MINP,IFREQ,IOBFR
      INTEGER TAPE,GROUP,Q
      DATA TDENT(1)(8HTRANSFOR)
      DATA TDENT(2)(8HMATION X)
      DATA TDENT(3)(8H TO     )
      DATA(TADD(I),I=1,11)(8HX,       ,8HLN X,   ,8HLN(LN(X),8HLN(1+X),,8
     1HLN(1+LN(,8HSQRT(X),,8H1/X,     ,8H1/(1+X),,8HARCSIN(X,8H2*ARCSIN,8
     2HARCSIN( )
      DATA(TADD2(I),I=1,11)(8H        ,8H        ,8H),      ,8H        ,
     18H1+X)),  ,8H        ,8H        ,8H        ,8H),      ,8H(SQRT(X),
     28HSQRT(X)))
      DATA PLUS(8H,       )
      IF(IGO-10)103,104,103
  104 TDENT(6)=BOOL(TADD2(3))
      GO TO 107
  103 IF(IGO-11)105,106,105
  106 TDENT(6)=BOOL(PLUS)
      GO TO 107
  105 TDENT(6)=BOOL(TADD2(1))
  107 TDENT(4)=BOOL(TADD(IGO))
      TDENT(5)=BOOL(TADD2(IGO))
      IGG=1
      REWIND 10
      REWIND 12
      NOG=NOJOBS
      SUM2P=0.0
      FNOBSP=0.0
      IDFF=3
 1002 DO 1001 JJX=1,NOG
      GO TO (20,20,20,2001),GROUP
   20 READ(12)NOBS,INTOV,NOINT,(INTER(I),I=1,NOINT)
    1 FORMAT(16I5)
      READ(TAPE,100)IDENT
  100 FORMAT(10A8)
      READ(TAPE,FORM)(OBSERV(I),I=1,NOBS)
```

```
C COMPUTE STANDARD DEVIATION OF THE OBSERVATIONS
      NUINT=NOINT
      FNOBS=NOBS
      SUM=0.0
      SUM2=0.0
      DO 3 I=1,NOBS
      OBSERV(I)=TFUNC(OBSERV(I))
    3 SUM =SUM +OBSERV(I)
      MEAN=SUM/FNOBS
      DO 4 I=1,NOBS
      OBSERP(I)=OBSERV(I)-MEAN
    4 SUM2=SUM2+OBSERP(I)*OBSERP(I)
      SDEV=SQRT(SUM2/(FNOBS-1.))
      CALL MAXMIN(OBSERV,NOBS,MAX,MIN)
      GO TO(202,203,203,220),GROUP
  203 IF(JJX.GT.1)GO TO 208
      MAXP=MAX-MEAN
      MINP=MIN-MEAN
      GO T) 209
  208 MAXP=AMAX1(MAXP,MAX-MEAN)
      MINP=AMIN1(MINP,MIN-MEAN)
  209 SUM2P=SUM2P+SUM2
      FNOBSP=FNOBSP+FNOBS
      WRITE(12)(OBSERP(I),I=1,NOBS)
      GO TO(220,202,1001,220),GROUP
  204 MAX=MAXP
      MIN=MINP
      MEAN=0.0
      SDEV=SQRT(SUM2P/(FNOBSP-FLOAT(NOJOBS)))
  202 RANGE=MAX-MIN
       DO 1000 I=1,NUINT
      IF(GROUP.LT.4)GO TO 198
      INTERV=INTERP(I)
      GO TO 199
  198 INTERV=INTER(I)
  199 FNTERV=INTERV
      DELTA=RANGE/FNTERV
      STEP(1)=MIN+DELTA
      IFREQ(1)=0.0
      IFREQ(INTERV+1)=0.0
      DO 5 J=2,INTERV
      IFREQ(J)=0.0
    5 STEP(J)=STEP(J-1)+DELTA
      STEP(INTERV)=MAX
      GO TO(205,205,220,206),GROUP
  205 DO 6201 L=1,INTERV
 6201 STIP(L)=STEP(L)-MEAN
      DO 6 K=1,NOBS
      JJ=(OBSERV(K)-MIN)/DELTA
    6 IFREQ(JJ+1)=IFREQ(JJ+1)+1.0
      IFREQ(INTERV)=IFREQ(INTERV)+IFREQ(INTERV+1)
      Q=Q+1
      PRINT 13,IDENT
      PRINT 7,Q,NOBS,INTERV,IDENT
    7 FORMAT(16H0CHI  SQUARE TEST,I4,5H WITH,I6,17H OBSERVATIONS AND,I5,1
     11H INTERVALS,,2A8,A4,3A8)
      CALL TRAN(STIP,IDEF,FNOBS)
      IF(I-1)97,99,97
   99 CALL PRINT1(JJX)
   97 PRINT 13,IDENT
```

```
 13 FORMAT(1H1,10A8)
    PRINT 7,Q,NOBS,INTERV,TDENT
    GO TO 97
206 PRINT 13,POOLID
    Q=Q+1
    PRINT 77,Q,FNOBSP,INTERV,TDENT
 77 FORMAT(16HOCHI SQUARE TEST,I4,5H WITH,F8.0,24H POOLED OBSERVATIONS
   1 AND,I5,11H INTERVALS.,2A8,A4,3A8)
    REWIND 12
    DO 368 K=1,NOJOBS
    NOBS=NOBSA(K)
    READ(12)(OBSERV(L),L=1,NOBS)
    GO TO (444,445),IPRINT
444 IF(I.EQ.1.AND.IGG.EQ.2)CALL PRINT1(K)
445 DO 66 IFG=1,NOBS
    JJ=(OBSERV(IFG)-MIN)/DELTA
 66 IFREQ(JJ+1)=IFREQ(JJ+1)+1.0
368 CONTINUE
    IFREQ(INTERV)=IFREQ(INTERV)+IFREQ(INTERV+1)
    IF(I.GT.1)GO TO 207
    GO TO (442,207),IPRINT
442 GO TO (207,447),IGG
447 PRINT 13,POOLID
    PRINT 77,Q,FNOBSP,INTERV,TDENT
207 CALL TRAN(STEP,IDFF,FNOBSP)
 97 PRINT 88,INTOV,INTERV
 88 FORMAT( 61H THE DEGREES OF FREEDOM WAS COMPUTED ON THE BASIS OF AT
   1 LEASTI5,40H OBSERVATIONS IN EACH OF A SUBSET OF THE,I5,11H INTERV
   2ALS.)
    PRINT 30,MEAN,SDEV
 30 FORMAT(6HOMEAN=,  E14.8,23H ...STANDARD DEVIATION=,E14.8)
    PRINT 8,RANGE,MAX,MIN,DELTA
  8 FORMAT(8HORANGE =,E15.8,12H   MAXIMUM =,E15.8,12H   MINIMUM =,E15.
   18,19H    INTERVAL WIDTH =,E15.8)
    PRINT 888
888 FORMAT(42H0      UPPER BOUND  FREQUENCY      BAR CHART,50X,35HCHI SQ
   1U CONTR    OBS FR      THEO FR)
    CALL MAXMIN(IFREQ,INTERV,XAM,SCALE)
    IF(XAM.GT.65.0)GO TO 91
    SCALE=1.0
    GO TO 92
 91 SCALE=XAM/65.0
 92 DO 9 K=1,INTERV
    IFGRPH=IFREQ(K)/SCALE+.5
    IF(IFGRPH.LE.0)GO TO 12
 11 DO 10 IFG=1,IFGRPH
 10 FGRAPH(IFG)=XXX
 12 IFGRPH=IFGRPH+1
    IF(IFGRPH-66)2033,2035,2035
2033 DO 2027 IFG=IFGRPH,65
2027 FGRAPH(IFG)=BLANK
2035 IF(CHS(K))2040,2041,2041
2040 PRINT 2029,STEP(K),IFREQ(K),FGRAPH
    GO TO 9
2041 PRINT 2029,STEP(K),IFREQ(K),FGRAPH,CHS(K),IOBFR(K),CMPFR(K)
2029 FORMAT(2X,F15.4,2X,F7.0,4X,1HI,65A1,1X,F8.3,1X,F10.1,2X,F10.3)
  9 CONTINUE
    PRINT 694,SCALE
694 FORMAT(13HOX REPRESENTS,F10.4,14H OBSERVATIONS.)
    IF(IDF)2048,2048,2049
```

```
2048 PRINT 2050
2050 FORMAT(35HL  CHISQUARE COULD NOT BE COMPUTED.)
     GO TO 1000
2049 PRINT 2039,CHISUM,IDF
2039 FORMAT(12HOCHISQUARE =,F16.6,5H WITH,I4,20H DEGREES OF FREEDOM.)
1000 CONTINUE
     GO TO(1001,1001,1001,221),GROUP
1001 CONTINUE
     GO TO(221,223,222,221),GROUP
 222 IGG=2
 223 GROUP=4
     IDFF=NOJOBS+2
     NOG=1
     NUINT=NOINTP
     INTOV=INTOVP
     GO TO 1002
 220 STOP
 221 RETURN
     END
T          SUBTYPE,FORTRAN,LMAP,PRIN
     SUBROUTINE TRAN(STEP,IDFF,OB)
C THIS SUBROUTINE FITS A NORMAL CURVE WITH MEAN ZERO AND STANDARD
C DEVIATION SDEV TO THE DATA IN IFREQ WHERE THE UPPER BOUND OF EACH
C INTERVAL IS IN THE CORRESPONDING ENTRY IN STEP(I).  OB  IS THE TOTAL
C NUMBER OF OBSERVATIONS.THE ROUTINE GROUPS THE DATA SO THAT THERE ARE
C AT LEAST INTOV THEORETICAL VALUES PER INTERVAL AND THEN COMPUTES THE
C CHISQUARE STATISTIC(CHISUM) TO GIVE AN ESTIMATION OF THE GOODNESS OF
C FIT.ON EXIT FROM THE ROUTINE,IDF CONTAINS THE NUMBER OF DEGREES OF
C FREEDOM.CHI(J)=-1 IF THE JTH INTERVAL WAS NOT THE LAST OF A GROUP,
C OTHERWISE IT CONTAINS (OBSERVED FREQUENCY-THEORETICAL FREQUENCY)**2
C DIVIDED BY THEORETICAL FREQUENCY.
C IOBFR(J) IS     ,ON EXIT,THE OBSERVED FREQUENCY,IF THE JTH INTERVAL WAS
C THE LAST OF A GROUP,OTHERWISE ITS CONTENTS ARE MEANINGLESS.LIKEWISE,
C CMPFR(J) CONTAINS THE THEORETICAL FREQUENCIES.
      COMMON MAX,MIN,OBSERV(14000),NOBS,STIP(400),IFREQ(401),SDEV,IDF
      COMMON CHI(400),CHISUM,IOBFR(400),CMPFR(400)
      COMMON INTOV,FNOBS,INTER(100),FGRAPH(65),STAP(400),TAPE,IGO
      COMMON NOJOBS,FORM(10),POOLID(10),OBSERP(14000),INTERP(100)
      COMMON NOBSA(500),INTOVP,NOINTP,GROUP,INTERV,IPRINT
      DIMENSION STEP(400)
      REAL MAX,MIN,IFREQ,IOBFR
      FNTOV=INTOV
      FOT=0.0
      KOUNT=0
      CHISUM=0.0
      PROBO=0.0
      FO=0.0
      INTM1=INTERV-1
    3 DO 10 J=1,INTERV
      IF(J.NE.INTERV)GO TO 1
      PROBN=1.0
      GO TO 2
    1 PROBN=FREQ(STEP(J)/SDEV)
    2 FOC=OB*(PROBN-PROBO)
      FO=IFREQ(J)+FO
      IF(FOC-FNTOV)4,4,5
    4 CHI(J)=-1.0
      GO TO 10
    5 FOT=FOT+FO
      REMAIN=OB*(1.0-PROBO)
```

```
      IF(OB*(1.0-PROBN)-FNTOV)6,6,9
    6 FO=FO+(OB-FOT)
      FOMRE=FO- REMAIN
      CHI(INTERV)=FOMRE*FOMRE/REMAIN
      IOBFR(INTERV)=FO
      CMPFR(INTERV)=REMAIN
      CHISUM=CHISUM+CHI(INTERV)
      KOUNT=KOUNT+1
      JJ=J
      GO TO 12
    9 FOMFOC=FO-FOC
      CHI(J)=FOMFOC*FOMFOC/FOC
      CHISUM=CHISUM+CHI(J)
      IOBFR(J)=FO
      CMPFR(J)=FOC
      KOUNT=KOUNT+1
      FO=0.0
      PROBO=PROBN
   10 CONTINUE
      GO TO 17
   12 IF(JJ-INTM1)14,14,17
   14 DO 16 J=JJ,INTM1
   16 CHI(J)=-1.0
   17 IDF=KOUNT-IDFF
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      SUBROUTINE MAXMIN(A,K,AMAX,AMIN)
      DIMENSION A(14000)
      AMAX=A(1)
      AMIN=A(1)
      DO 900 I=2,K
      IF(AMAX-A(I))1,5,5
    1 AMAX=A(I)
      GO TO 900
    5 IF(AMIN-A(I))900,900,3
    3 AMIN=A(I)
  900 CONTINUE
  220 RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      SUBROUTINE PRINT1(K)
      COMMON MAX,MIN,OBSERV(14000),NOBS,STIP(400),IFREQ(401),SDEV,IDF
      COMMON CHS(400),CHISUM,IOBFR(400),CMPFR(400)
      COMMON INTOV,FNOBS,INTER(100),FGRAPH(65),STEP(400),TAPE,IGO
      COMMON NOJOBS,FORM(10),POOLIC(10),OBSERP(14000),INTERP(100)
      COMMON NOBSA(500),INTOVP,NOINTP,GROUP,INTERV,IPRINT
      PRINT 91,K
   91 FORMAT(44H0A LISTING OF THE OBSERVATIONS IN SAMPLE NO.I4,9H FOLLOW
     1S./1H0)
      LC=0
   98 DO 92 LL=1,NOBS,8
      LC=LC+1
      KM=LL+7
      IF(NOBS-KM)41,95,95
   41 KM=NOBS
   95 PRINT 93,LC,(OBSERV(L),L=LL,KM)
   93 FORMAT(1H ,I5,3H)   ,8E15.8)
   92 CONTINUE
      RETURN
```

```
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T1(X)
      T1=X
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T2(X)
      T2=ALOG(X)
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T3(X)
      T3=ALOG(ALOG(X))
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T4(X)
      T4=ALOG(1.0+X)
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T5(X)
      T5=ALOG(1.0+ALOG(1.0+X))
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T6(X)
      T6=SQRT(X)
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T7(X)
      T7=1.0/X
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T8(X)
      T8=1.0/(1.0+X)
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T9(X)
      T9=ATAN(X/SQRT(1.0-X*X))
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T10(X)
      T10=2.0*ATAN(SQRT(X)/SQRT(1.0-X))
      RETURN
      END
T         SUBTYPE,FORTRAN,LMAP,PBIN
      FUNCTION T11(X)
      T11=ATAN(SQRT(X)/SQRT(1.0-X))
      RETURN
      END
```

VIII. REFERENCES

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